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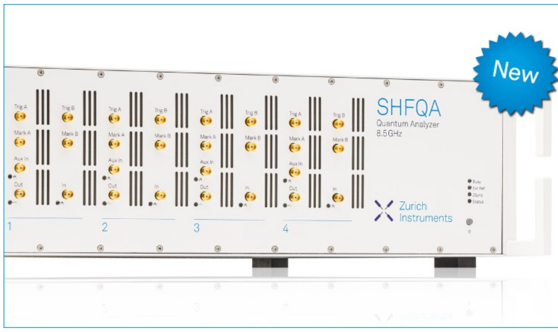

On Well-posedness of Source Identification Elliptic Problem with Nonlocal Boundary Conditions

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Abstract. We study the well-posedness of the source identification problem for the two dimensional elliptic differential equation with nonlocal boundary conditions. Applying operator approaches, the exact estimates for the solution of this problem in Hölder norms are established.

INTRODUCTION

Theoretical aspects and methods of solutions of source identification problems for partial differential equations have been extensively investigated by many researchers (see [3, 4, 8, 9, 10, 11, 14, 15, 17, 18, 19] and the bibliography herein). Well-posedness of non classical boundary value problems for various differential and difference equations were investigated in a number of publications (see [1-22] and references therein).

In this paper, we study the source identification problem for the two dimensional elliptic differential equation with nonlocal boundary conditions

$$\begin{cases} -\frac{\partial^2 u(y,x)}{\partial y^2} - a(x)\frac{\partial^2 u(y,x)}{\partial x^2} + \delta u(y,x) = f(y,x) + p(x), \\ 0 < y < T, 0 < x < l, \\ u(0,x) = u(T,x), u_y(0,x) = u_y(T,x),\ u(\lambda,x) = \xi(x), 0 \le x \le l, \\ u(y,0) = u(y,l),\ \ u_x(y,0) = u_x(y,l),\ \ 0 \le y \le T, \end{cases} \tag{1}$$

where $a(x)$, $\xi(x)$ and $f(y,x)$ are given sufficiently smooth functions and $a(x) > 0,\ 0 < \lambda < T, \delta > 0$ is a sufficiently large number. Assume that all compatibility conditions are satisfied.

The well-posedness of the source identification problem (1) for the two dimensional elliptic differential equation with nonlocal boundary conditions. Applying operator approaches, the exact estimates for the solution of this problem in Hölder norms are established.

THE MAIN THEOREM ON WELL-POSEDNESS OF PROBLEM (1)

We introduce the Banach spaces $C^\beta[0,l]$ $(0 < \beta < 1)$ of all continuous functions $\varphi(x)$ satisfying a Hölder condition for which the following norms are finite

$$\| \varphi \|_{C^\beta[0,l]} = \| \varphi \|_{C[0,l]} + \sup_{0 \le x < x+\tau \le l} \frac{|\varphi(x+\tau) - \varphi(x)|}{\tau^\beta},$$

where $C[0,l]$ is the space of the all continuous functions $\varphi(x)$ defined on $[0,l]$ with the usual norm

$$\| \varphi \|_{C[0,l]} = \max_{0 \le x \le l} |\varphi(x)|.$$

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Theorem 1 *For the solution of the source identification problem (1) the following stability and coercive stability estimates hold:*

$$\|u\|_{C(C^{\beta}[0,l])} \leq M(\beta)\left[\|\xi\|_{C^{\beta}[0,l]} + \|f\|_{C(C^{\beta}[0,l])}\right],$$

$$\|u\|_{C_{0T}^{2+\alpha,\alpha}(C^{\beta}[0,l])} + \|u\|_{C_{0T}^{\alpha,\alpha}(C^{\beta+2}[0,l])} + \|p\|_{C^{\beta}[0,l]}$$

$$\leq \frac{M(\beta)}{\alpha(1-\alpha)}\|f\|_{C_{0T}^{\alpha,\alpha}(C^{\beta}[0,l])} + M(\beta)\|\xi\|_{C^{\beta+2}[0,l]},$$

where $M(\beta)$ is independent of α, $\xi(x)$ and $f(t,x), 0<\alpha<1,\ 0<\beta<1$.Here $C_{0T}^{\alpha,\alpha}(E)$ $(0<\alpha<1)$ is the Banach space obtained by completion of the set of $E-$valued smooth functions $\varphi(t)$ defined on $[0,T]$ with values in E in the norm

$$\|\varphi\|_{C_{0T}^{\alpha,\alpha}(E)} = \|\varphi\|_{C(E)} + \sup_{0\leq t<t+\tau\leq T} \tau^{-\alpha}(T-t)^{\alpha}(t+\tau)^{\alpha}\|\varphi(t+\tau)-\varphi(t)\|_{E},$$

where $C(E)$ stands for the Banach space of all continuous functions $\varphi(t)$ defined on $[0,T]$ with values in E equipped with the norm

$$\|\varphi\|_{C(E)} = \max_{0\leq t\leq T}\|\varphi(t)\|_{E}.$$

Proof. It is known that the differential expression

$$Av(x) = -a(x)v''(x) + \delta v(x) \tag{2}$$

define a positive operator A acting in $C^{\beta}[0,l]$ with domain $C^{\beta+2}[0,l]$ and satisfying the conditions $v(0)=v(l),\ v_x(0)=v_x(l)$. Therefore, source identification problem (1) can be written in abstract form

$$\begin{cases} -u''(t) + Au(t) = f(t) + p,\ \ 0<t<T, \\ u(0)=u(T),\ u'(0)=u'(T), u(\lambda)=\xi \end{cases} \tag{3}$$

in a Banach space E with unknown parameter $p=p(x)$ and unknown abstract function $u(t)=u(t,x)$. Here, element $\xi=\xi(x)$ and smooth abstract function $f(t)=f(t,x)$ defined on $[0,T]$ with values in E are given. Therefore, the proof of Theorem 1 is based on the positivity of the elliptic operator A in $C^{\beta}[0,l]$ [7] and the following theorem on well-posedness of problem (3).

Theorem 2 *Assume that $\xi\in D(A)$ and $f(t)\in C_{01}^{\alpha,\alpha}(E), 0<\alpha<1$. For the solution $\{u(t),p\}$ of source identification problem (3) in $C_{0T}^{\alpha,\alpha}(E)\times E$ the stability and coercive inequality*

$$\|u\|_{C(E)} \leq M\left[\|\xi\|_{E} + \|f\|_{C(E)}\right],$$

$$\|u''\|_{C_{0T}^{\alpha,\alpha}(E)} + \|Au\|_{C_{0T}^{\alpha,\alpha}(E)} + \|p\|_{E} \leq M[\|A\xi\|_{E} + \frac{1}{\alpha(1-\alpha)}\|f\|_{C_{0T}^{\alpha,\alpha}(E)}]$$

are satisfied, where M is independent of α, ξ and $f(t)$.

CONCLUSION

In the current work, the well-posedness of the source identification problem for the two dimensional elliptic differential equation with nonlocal boundary conditions is investigated. The exact estimates for the solution of this problem in Hölder norms are established. In future investigation, absolute stable difference schemes for approximately solution of the source identification problem for the two dimensional elliptic differential equation with nonlocal boundary conditions will be constructed and investigated.

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