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A Stochastic Resonator to Detect BPAM Signals; Analysis, PSR Desings and Sine-induced SR

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Abstract: A Stochastic Resonator has been considered as an alternative signal processing tool because of its noise-induced performance enhancement ability. Here, the resonator parameters, steady states and transition time of the system are redefined for BPAM signals such that the region in which the resonator benefits from noise can be identified. Simple parameter-induced stochastic resonance (PSR) designs are then built, based on this analysis in order to configure the resonator in the optimum region. Furthermore, Sine-induced SR based on using a periodic signal instead of noise is introduced to enhance the system performance and compared with noise-enhanced SR (NSR). It is shown that Sine-induced SR provides a performance enhancement as it needs less power and does not require an adjustment relevant to the background noise. The results indicate that a resonator improves the receiver performance by eliminating noise if its parameters and BPAM characteristics are set accurately as given in the PSR designs, otherwise the resonator can benefit from either a noise as in NSR, or a sine wave as proposed.

1. Introduction

Existing methods at mitigating noise may be ineffective at low Signal to Noise Ratios (SNRs), whilst also being inapplicable at the nanoscale in terms of the signal processing and energy requirements [1]. Stochastic Resonance (SR), is an alternative signal detection technique [2, 3], where, as supposed to removing noise, one uses noise to enhance the system performance. The term was first coined by Benzi *et al.* in 1981 [4, 5] and since then, SR has been observed in numerous systems across many fields [6, 7, 8]. As more systems were seen to exhibit SR, several generalized theories on the subject have thus been presented [9, 10, 11, 12, 13], but essentially, SR means that the system performance is enhanced by the means of noise.

The performance of the systems exhibiting SR has a peak at a non-zero noise intensity. In the literature, there are two methods used to enhance the performance, namely, noise-enhanced SR (NSR), and parameter induced SR (PSR). In NSR, defining the type of resonant noise and its correlation time, the effectual intensity is determined [14, 15, 16]. If an adjustment is required, NSR needs the knowledge of the background noise, which can be obtained by adaptive search methods, and this makes the system more complex. Moreover, the power consumption of a noise source can be another problem. In PSR, the system parameters are determined for the optimum performance [17, 18, 19] and intrinsically differ from one type of input signals to another. Whilst there are many studies on weak periodic input signals, there is not any basic and precise design methodology for a bipolar binary pulse amplitude modulated (BPAM) signal which is curious as it is one of the fundamental communications signalling motifs. In addition to this, no research concerning the use of a deterministic (as supposed to noise) resonant signal could be found.

Therefore, in this paper, the aim is to design a basic stochastic resonator for detecting BPAM signals and to subsequently increase the system performance by using a deterministic periodic resonant signal as supposed to noise. Since existing designs do not focus on the input signal, we define new parameters derived from the desired output and build a new stochastic resonator design based on the characteristic of the BPAM signal. The unique relation between the output and the system parameters is also clarified, and as a consequence, PSR methods are simplified. Then, a periodic signal is introduced as a resonant signal. It is found that the use of a periodic signal ensures a significant performance enhancement without tuning. To summarize, parameters defined to analyse the SR system with Langevin equation, a further performance enhancement in NSR by a sine wave, and simple desings for PSR are the key contributions of this paper.

This paper is organized as follows. In Sec. 2, the theory behind the stochastic resonator and its implementation is discussed and the method used to determine the design parameters explained and subsequently evaluated. The optimum configuration providing low BERs at low SNRs is also presented. Sec. 3 introduces the generalised analysis allowing the reader to identify any resonator configuration. Based on this analysis, simple PSR designs are also provided. In Sec. 4, the use of a resonator is taken further with the notion of using NSR and Sine-induced SR. Finally, the paper is concluded in Sec. 5.

2. The Stochastic Resonator

The stochastic resonator, in this paper, is a form of an overdamped Brownian motion in the symmetric quartic bistable potential [20, 21, 22, 23, 24]. It is modelled as a feedback system in which the forward loop consists of an arbitrary amplifier with gain M followed by an integrator as shown in Fig. 1. The feedback loop consists of two amplifiers with gains a and b and two multipliers.

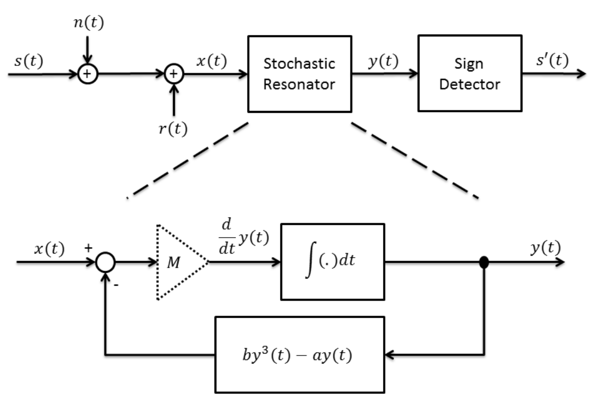

Fig. 1: *The BPAM receiver*

Defining the transmitted signal, $s(t)$, which is typically corrupted by AWGN, $n(t)$, the input of the stochastic resonator, $x(t)$, is given by

$$x(t) = s(t) + n(t) + r(t), \tag{1}$$

where $r(t)$ is an intentionally added resonant signal under the control of the system designer.

The integrator is based on the *midpoint rule* approximation which keeps the previous output, y_p, and the previous derivative, y'_p, in its memory. The output, $y(t)$ is thus

$$y(t) = y_p + \frac{y'_a + y'_p}{2} \times t_s, \tag{2}$$

where here, to illustrate the system working as close to a continuous system as possible, $t_s = T_b/100$, and T_b is the bit interval of $s(t)$. Therefore, within this model there are three adjustable parameters, a, b and M, together with one controllable signal, $r(t)$, all of which will affect the output to the stochastic resonator, $y(t)$.

In order to achieve the aims of this work, the following process is required. Firstly, in the absence of both $n(t)$ and $r(t)$, the steady state behaviour shall be analysed to determine the bounds of a and b. This is followed by incorporating knowledge of the systems transient behaviour such that a and b can be refined (if necessary) and M can be determined. The result of this section therefore is that, given some knowledge of $s(t)$, and still in the absence of $n(t)$ and $r(t)$, the system should operate without an error. It is then possible to begin the analysis into how $n(t)$ affects the performance and subsequently how $r(t)$ can be controlled to mitigate its effects.

2.1. Steady State Behaviours

Referring to Fig. 1, under the assumption that $M = 1$ for the duration of the steady state behaviour analysis, the resonator's output is derived from

$$\frac{dy}{dt} = ay - by^3 + x, \tag{3}$$

where $x(t)$ is only $s(t)$ and $s(t)$ follows a NRZ BPAM scheme, which can take only the values of either A or $-A$ during a bit interval T_b. For $s'(t) = sign(y(t))$ to have the same polarity as $s(t)$ does, a and b must be chosen carefully.

First of all, the resonator applies a barrier to the input. It can be calculated by determining the local maxima (or minima) of dy/dt where $x(t) = 0$, and be given by $h_b = \sqrt{4a^3/27b}$. $x(t)$ forces output to have the same signs when $A > h_b$. This is the first relationship between a, b and the magnitude of the input, A, for error free operation.

To explain this, consider Fig. 2. If $x(t) = A \geq h_b$, i.e a positive BPAM symbol greater than the barrier height, regardless of value of $y(t)$ at the same instant, $y(t)$ will settle to a positive value as required. If $x(t) = A < h_b$, i.e a positive BPAM symbol but with a magnitude lower than the barrier height, depending upon the value of $y(t)$ at the same instant, $y(t)$ may settle to the positive value, correctly, or the negative value incorrectly.

Secondly, it is needed to known exactly what $y(t)$ will be. Although making the barrier smaller than the input is enough for the polarity issue, later, it is going to be shown that the steady state matters when the bit interval is taken into account. By definition, the roots of (3) are the steady states, and there is only one real root if $A > h_b$. As is typical within any system that the output has a gain (or loss), k, the steady state can be given by kA where $x(t) = A$. If so, it must satisfy (3), which reduces to $0 = akA - bk^3A^3 + A$. Therefore, $A > h_b$ can be rearranged as $\sqrt{(ak+1)/(bk^3)} > \sqrt{(4a^3)/(27b)}$, which dictates that the parameter a times the gain k must be smaller than 3, $ak < 3$. And, the parameter b is determined by $(ak+1)/(A^2k^3)$.

As a result, the resonator parameters (a, b) are determined to make output with a desired signs and gain, k. It is based on the steady states of (3) with the assumption of having a sufficient time

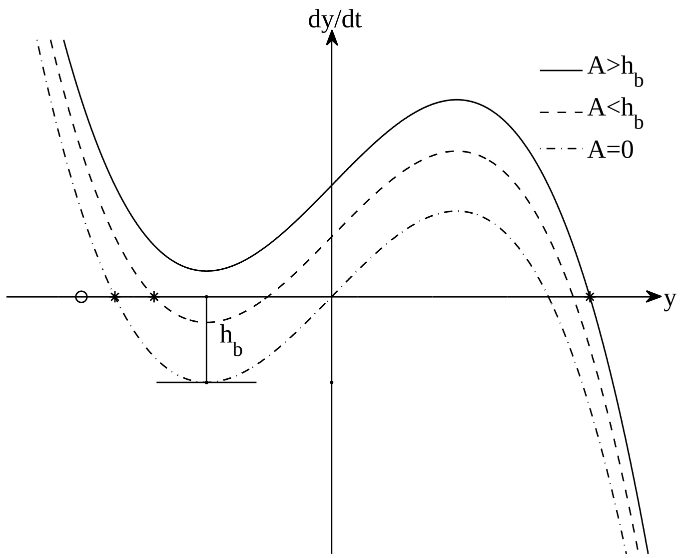

Fig. 2: *The derivatives of output where the output of the system is derived by Langevin Equation, the noise is absent, and* $x(t) = A$.

for the output to become stable. However, the input is stable only for a period of time T_b, so the output may not reach the steady state within the same period.

2.2. Transient Behaviours

As has been discussed so far, upon an input change, the output $y(t)$ needs a finite time to transient from $-kA$ to kA when input $x(t) = A$. It is called the *transition time* t_r, and parameters (a, b) must be determined to satisfy $t_r < T_b$.

The transition time t_r can be derived from the integral of one over the derivative of output within the interval $[-kA, kA)$. Assuming that $y = -kA$ and $x(t) = -A$ at $t < 0$ and $x(t) = A$ for $t \geq 0$. In such case, $y(t)$ starts increasing according to its derivative. Then, t_r is given by

$$t_r(y \lesssim kA) = \int_{-kA}^{y} \frac{dt}{dy} dy \tag{4}$$

It is preferred to use y as an upper boundary from which the transition shape can be obtained. In addition to that, the output never reaches kA theoretically. Due to the fact while y is getting closer to kA, dt/dy goes infinite. Therefore, the upper boundary should be smaller than kA so it is set to $y = 0.99 \times kA$ which means the system is working to a 1% steady state error.

The derivation of (4) is provided in the Appendix, and it can be simplified as $t_r = kt_n(ak, kA)$ where t_n is the normalized transition time, a function of ak and kA. Fig. 3 illustrates the effects of kA and ak on t_n, and it is obvious that t_n goes to infinite while ak is getting closer to 3. Another point to note is that the effect of kA on the normalized transition length is not significant when compared to ak.

As the input signal has a bit interval, T_b, kt_n must be smaller than T_b in order the output to transient and settle down before the following bit comes, such that $kt_n < T_b$. Additionally, from Fig. 3, t_n is always greater than 2, which result in $2 < t_n < T_b/k$. However, the relation between

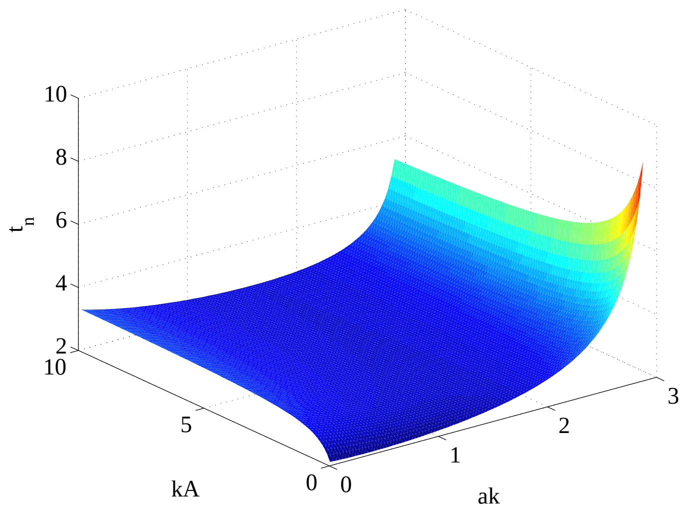

Fig. 3: *The effects of ak and kA on the normalize transition time, $t_n(y)$.*

k and the bit interval T_b causes an attenuation problem because k has been used for determining output steady state. For example, if the receiver is designed for BPAM signal with $T_b = 1ns$, then k has to be smaller than 5×10^{-10}. Such attenuation is not practical.

The attenuation problem can be solved by the parameter M in Fig. 1. Considering that t_r has k multiplier, the derivative of the output must have $1/k$ multiplier, and it does, which is obvious when (3) is re-written as

$$\frac{dy}{dt} = M\frac{1}{k}\left(aky - \frac{ak+1}{(kA)^2}y^3 + kA\right). \tag{5}$$

In (3), M is introduced to eliminate the effect of k on t_r and to have a control on the frequency response. It can be given by

$$M = m \times \frac{k}{T_b}, \tag{6}$$

where m is related to the normalized transition time t_n. That must be greater than t_n but does not have to equal to t_n so it is preferred to use different notations.

As a result, the new parameters used to define the resonator are kA for output steady state, ak for barrier height, and m for frequency response. A resonator can be analysed and designed by only two of them as shown next.

2.3. Parameter Choices

The choice of parameters is critically important for the output performance of the resonator. First, an example with significant parameters is illustrated and then the optimum values based on the output performance is obtained.

The example is a resonator design with the parameters $ak = 1.5$ and $m = 3$. Given the input characteristics and desired output;

$$T_b = 10^{-9}\text{s}, \; A = 1, \; y_{\text{desired}} = 1,$$

The design thus has the variables;

$$
\begin{aligned}
k &= y_{\text{desired}}/A = 1, \\
a &= ak/k = 1.5, \\
b &= (ak+1)/(A^2k^3) = 2.5, \\
M &= (mk)/(T_b) = 3 \times 10^9.
\end{aligned}
$$

The output of the stochastic resonator with these parameters can complete the transition in a bit interval time as illustrated in Fig. 4 (a) so that it can follow the input sign. When m is set to 1 which is smaller than the minimum normalized transition time $t_n(ak = 1.5, kA = 1) \simeq 2.46$, a bit interval is not long enough to complete the transition as illustrated in Fig. 4 (b). When $ak \geq 3$, $h_b \geq A$ so that the output sticks in either positive or negative side. Fig. 4 (c) illustrates the case where $ak = 3$. It is clear that, because of the barrier height h_b, the output does not change its sign while the input has both negative and positive values. As a result, in terms of design, there are two significant parameters with simple inequalities $0 \leq ak < 3$ and $m \geq t_n(ak, kA)$. However, these inequalities do not specify the optimum values of the parameters where the background noise is present.

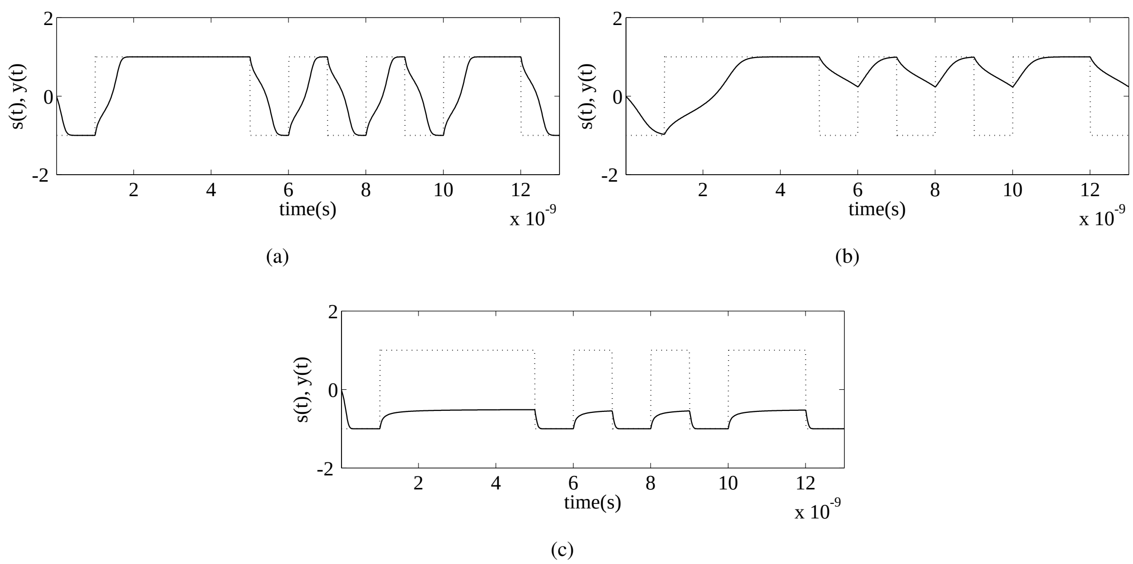

Fig. 4: *The output of stochastic resonators with* ($ak = 1.5, m = 3$)*,* ($ak = 1.5, m = 1$)*, and* ($ak = 3, m = 3$) *respectively. Note that* $k = 1$*, input is* $s(t)$ *with dash line, and output is* $y(t)$ *with solid line.*

When the signal is weak, the presence of the background noise is supposed to increase the performance. This is valid for $ak \geq 3$. However, as Fig. 5 indicates, if the signal is not weak ($ak < 3$), the background noise and the barrier h_b decrease the performance. Therefore, ak must be smaller than 1.

The other parameter m is directly related to the input signal frequency 1/Tb, and Fig. 6 indicates that $m = 3$ is almost optimum. When m is greater than that, the BER curve moves through the higher SNRs. When m is 2 and 1, the curve is shaped so that the BER slightly decreases at lower SNRs, but significantly increases at higher SNRs. Therefore, to have low BERs in general, m must be set to 3.

The stochastic resonator, whose output is given by (3), has been designed in a way different from those in the literature. It is based on the generalization of the parameters in terms of the signal

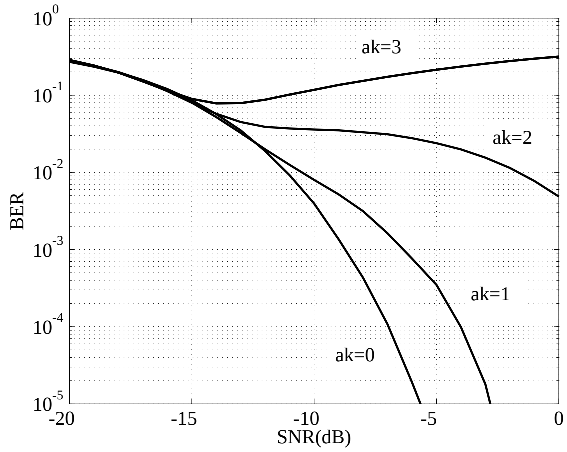

Fig. 5: *BERs where* $m = 3$, $ak = 0 : 3$, $x(t) = s(t) + n(t)$ *and* $n(t) \sim N(0, \sigma_n^2)$

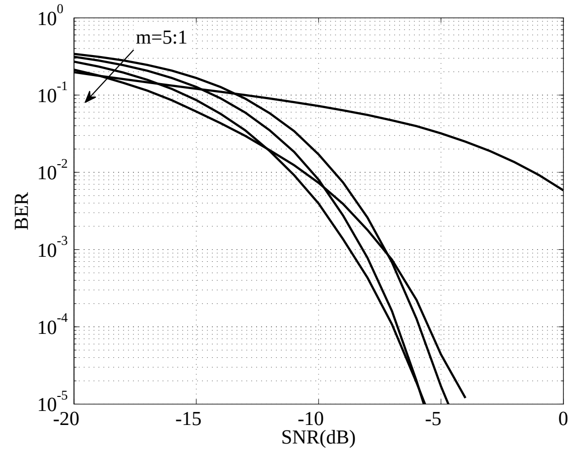

Fig. 6: *BERs where* $ak = 0$, $m = 1 : 5$, $x(t) = s(t) + n(t)$ *and* $n(t) \sim N(0, \sigma_n^2)$

characteristics. It provides a better understanding on the relations between the signal amplitude and the frequency response of the system. New parameters k, ak, kA, t_n, m and M are defined. It is emphasized that m and ak are the most significant parameters. They are used to determine the frequency response of the system and the barrier to the input signal amplitude. Finally, the optimum values of m and ak are specified by the means of the output performance. The application of this design on PSR method and further analysis are discussed in the next section.

3. Analysis and Design for PSR

The parameters defined in the section 2. are used to analyse the stochastic resonator whose response depends on not only the bit interval but also the amplitude of the input signal. Although well-known transformations cannot be applied because of that, the new interchangeable parameters which represent those; a, b, M, A and T_b together help us to analyse and design the resonator.

Given a, b, M, A and T_b, parameters ak and m can be determined. First, to find out ak, the barrier obtained from (3) must be equal to the barrier derived by A and ak; $h_b = \sqrt{4a^3/27b} = \sqrt{4(ak)^3A^2/27(ak+1)}$. Then, ak and normalized A can be given by

$$A = \sqrt{\frac{a^3(ak+1)}{b(ak)^3}} = \sqrt{\frac{a^3}{b}} A_n, \tag{7}$$

where

$$A_n = \sqrt{\frac{(ak+1)}{(ak)^3}}. \tag{8}$$

Secondly, to determine m and normalized T_b, ak in (8) and a are used in (6) as

$$m = MaT_b/ak = T_{bn}/ak, \tag{9}$$

where

$$T_{bn} = MaT_b. \tag{10}$$

These two normalizations have only one precise solution and bring out a fundamental analysis of a resonator. Fig. 7 is created by these normalizations. It can be used to determine m and ak parameters indicating the region in which resonator works. For example, if $A_n = 10$ and $T_{bn} = 100$, $ak \ll 1$ and $m \gg 3$. So the resonator works in the region where SR effect can not be observed ($ak \ll 1$), and BER is low at high SNRs ($m \gg 3$). Fig. 7 is more practical when compared to (8).

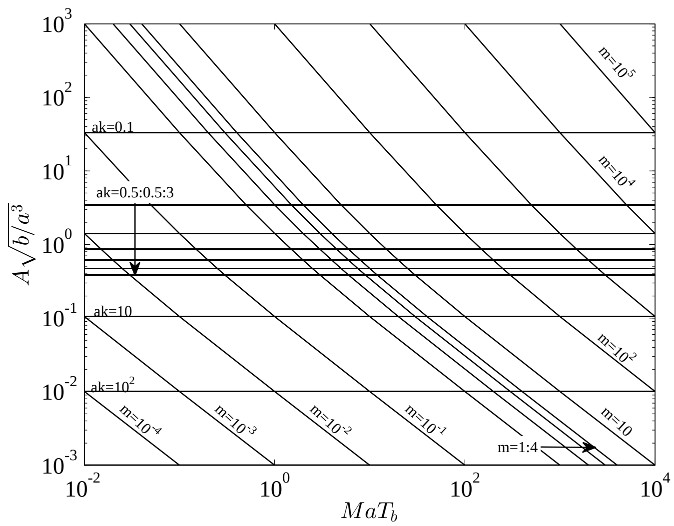

Fig. 7: *Normalized amplitude and bit interval with corresponding m and ak parameters*

Similar to this analysis, PSR designs can also be simplified. Essentially, there are two possible tuning methods; the first one, PSR-1, is determining the optimum a, b, and M for a given A and T_b. The second one, PSR-2, is determining the optimum input signal characteristics A and T_b where a, b, and M are given. The M parameter is set to 1 for the following designs to have more general expressions. As stated in Sec. 2.3, the variables for optimum BER performance must be chosen as $m = 3$ and $ak \ll 1$. For PSR-1, the resonator can be designed by

$$\begin{aligned} k &= T_b/m, \\ a &= ak/k, \\ b &= (ak+1)/(A^2k^3). \end{aligned} \tag{11}$$

For PSR-2, A and T_b are determined by (7) and (9) as

$$\begin{aligned} k &= ak/a, \\ T_b &= m \times k, \\ A &= \sqrt{\frac{a^3(ak+1)}{b(ak)^3}}. \end{aligned} \tag{12}$$

An analysis for a stochastic resonator and PSR designs have been given in this section. The analysis is based on determining m and ak by the normalized amplitude and bit interval of the signal applied to the resonator. PSR methods are established in (11) and (12). Setting $m = 3$ and $ak \ll 1$ is sufficient to have an optimum BER performance for these designs. Although PSRs are simplified and provide significant performance improvement, they may not be applicable in some circumstances. When PSR is not available, another method, NSR, can be applied as discussed in Sec. 4.

4. NSR and Sine-induced SR

When the input signal, $s(t)$, is weak according to the system barriers, the output can not transient. In this case, the resonant signal $r(t)$ can help the input to exceed the system barriers and improve the system performance. The resonant signal can be either a type of noise as in NSR method or a periodic wave as indicated in this section. These two resonant signals are going to be compared in terms of BER performances and power requirements.

NSR can be used when the parameters and the input characteristics are chosen to have a weak input signal. The PSR-1 design method of Sec. 3 is used to build the resonator to demonstrate an NSR application. Here, the input is a BPAM signal with $A = \sqrt{4/27}$ and $T_b = 9$. Critical parameters are set as $m = 3$ and $ak = 3$ to observe SR effect, and as a result, resonator parameters are $a = 1$ and $b = 1$. If there is only a BPAM signal at the input, the resonator output will show either positive or negative values depending on the first bit received as in Fig. 8 (a). When an AWGN resonant noise with an optimum power (BER is minimum at SNR, $20\log(A/\sigma) \cong -14$ dB in Fig. 5) is added to the input, the output starts to transient along with BPAM signal as shown in Fig. 8 (b).

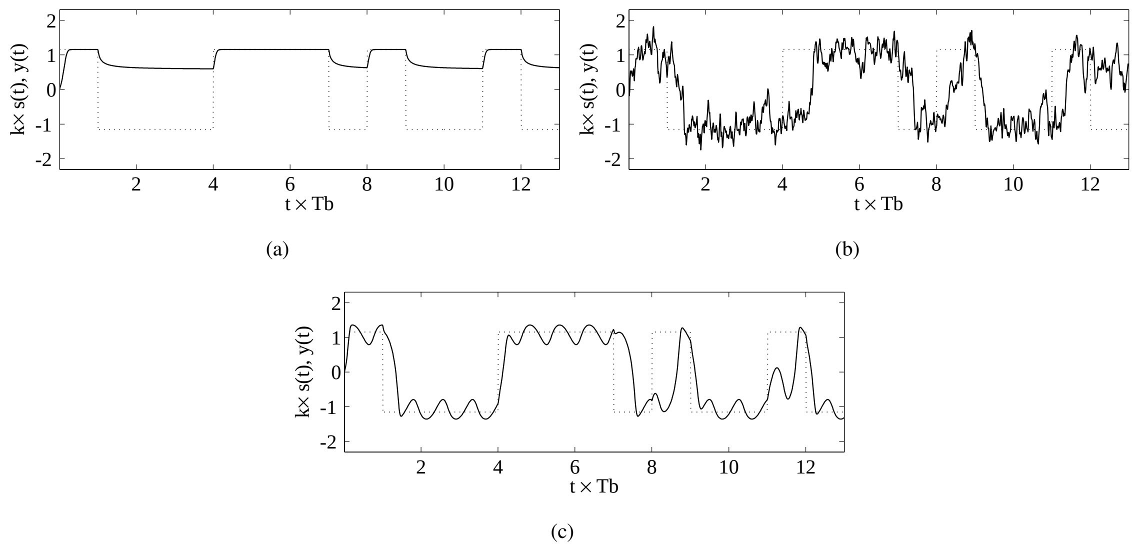

Fig. 8: *Outputs of the stochastic resonator with the parameters;* $a = 1, b = 1$ *and* $s(t)$ *is BPAM with* $A = \sqrt{4/27}$ *and* $T_b = 9$*. (a)* $x(t) = s(t)$*. (b)* $x(t) = s(t) + r(t)$ *where* $r(t) \sim N(\mu = 0, \sigma_{opt} = 1.93)$*. (c)* $r(t) = 2A\sin(2\pi(1.3/T_b)t)$*.*

However, with this method, NSR, suffers from some drawbacks. For instance, when background noise $n(t)$ is present, the resonant noise $r(t)$ has to be re-adjusted to have the optimum SNR. Although it can be easily determined by $\sigma^2_{opt} = \sigma^2_{n(t)} + \sigma^2_{r(t)}$, background noise power must be known. Even if $\sigma^2_{n(t)}$ can be measured, when $\sigma_{n(t)}$ is already higher than σ_{opt}, injecting any resonant noise $r(t)$ will cause a decrease in performance. Besides these drawbacks, as a source, resonant noise may demand high power, especially when background noise power is much less than the optimum one.

A periodic signal is suggested as an alternative to resonant noise. For the sake of simplicity, it is considered as a sine wave, $B\sin(2\pi ft)$, which can also help the input to exceed the system barrier as shown in Fig. 8 (c). It is aimed to detect effectual sine waves providing a significant performance improvement. BER performance results for various sine waves are illustrated in Fig. 9 where $\sigma_{n(t)} = 0$ and $\sigma_{n(t)} = 0.68$ respectively. When $\sigma_{n(t)} = 0$, there are many sine waves

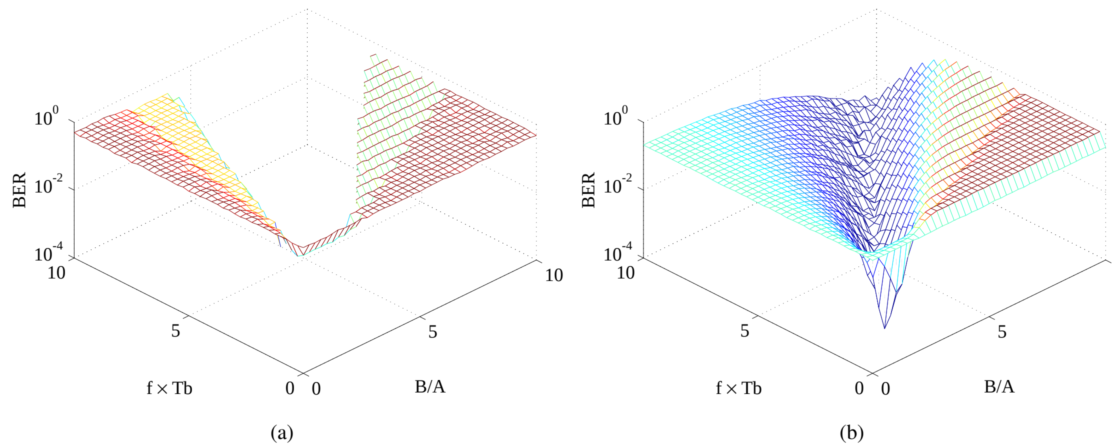

Fig. 9: *BERs where $r(t)$ is $B\sin(2\pi ft)$, and $\sigma_{n(t)} = 0$ and 0.68 respectively.*

providing error free output signal of which performance cannot be plotted in log scale, and there is a pattern restricting the amplitudes, B, and the frequencies, f. Such as in Fig. 9 (b), this pattern is narrowed by the background noise ($\sigma_{n(t)} = 0.68$), thus, output is not error free any more. For this example where $m = 3$, $ak = 3$ and SNR $= -5$ dB, a basic curve fitting is applied to find out the best f and B couples as illustrated in Fig. 10. The fitted curve for resonant sine waves can be given by

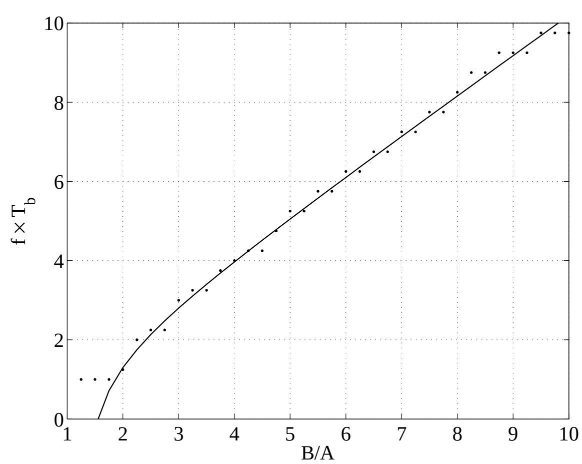

Fig. 10: *Frequency and Amplitude couples (dots) providing lowest BERs and fitted curve (solid) given in (13).*

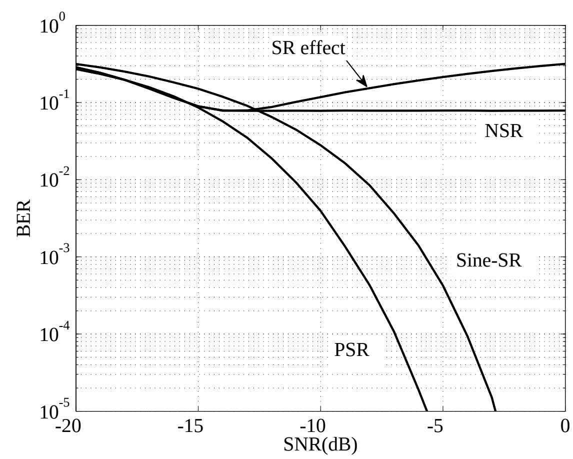

Fig. 11: *BERs where $x(t) = s(t) + n(t) + r(t)$ and $n(t) \sim N(0, \sigma_n^2)$. NSR performance with $r(t) \sim N(0, \sigma_r^2)$. Sine wave SR performance with $r(t) = 2A\ sin\ (2\pi(1.3/Tb)t)$.*

$$f \times T_b = \frac{1}{1 - B/A} + B/A + 0.3. \tag{13}$$

Choosing $B \geq 2A$ and $f \geq 1/Tb$, the BER decreases significantly. On the other hand, the power of the sine wave is basically $B^2/2$. To consume less power, B must be $2A$ and f is thus $1.3/T_b$.

Both NSR and sine wave SR can be compared in terms of performance and power issues. It is clear from Fig. 11 that NSR cannot enhance the performance further than that obtained by the optimum background noise. However, sine wave SR decreases the BER as much as a PSR method does. When it comes to power requirement, it is seen that resonant noise power, $\sigma^2_{r(t)}$, is between 0 and σ^2_{opt}, while the power of sine wave is always $B^2/2$. The minimum power consumption in sine wave SR is only $10\log((B^2/2)/(A^2)) = 3$ dB so that $\sigma^2_{r(t)}$ is always greater than $B^2/2$ where SNR $>$ SNR$_{opt} + 3$ dB. It can be concluded that whilst sine wave SR needs less energy and does not requires an adjustment depending on background noise, it provides an increase in performance.

5. Conclusions

A stochastic resonator can be used as an alternative signal processing tool at low SNRs and potentially applications where size and energy considerations are paramount. Owing to the expressions given for steady states and transition time, comprehension and application of this resonator becomes attainable. Furthermore, the analysis shown here simplifies the application of PSR methods. A resonator filters the background noise and thus, the system performance can be enhanced considerably by the means of only two parameters a and b. While digital filters are providing a similar performance with hundreds of coefficients, having only two coefficients may be the most significant advantages of the resonator. Finally, a sine wave is proposed as a resonant signal instead of noise where PSR methods are not available. The evidence presented thus far supports the idea that a sine wave supplies a better performances enhancement while decreasing the complexity and power consumption.

6. Appendices

To separate the roots of (3) where $x = A$, the right hand side can be rewritten as:

$$\frac{dy}{dt} = -b(y - kA)(y - r_1)(y - r_2), \tag{14}$$

where one root is kA and the $r_{1,2}$ is given by

$$r_{1,2} = kA\left(-\frac{1}{2} \pm i\sqrt{\frac{3}{4} - \frac{ak}{ak+1}}\right). \tag{15}$$

To be able to write down the expression for $t(y)$, p and q are introduced as

$$q = kA\sqrt{\frac{3}{4} - \frac{ak}{ak+1}}, \tag{16}$$

$$p = kA/2, \tag{17}$$

$$r_{1,2} = -p \pm iq. \tag{18}$$

(4) is thus

$$t(y) = \frac{1}{-b}\int_{-kA}^{y}\left(\frac{D}{(y-kA)} + \frac{E}{(y-r_1)} + \frac{E}{(y-r_2)}\right)dy, \tag{19}$$

where,

$$D = \left((kA)^2\left(2 - \frac{ak}{ak+1}\right) + kA\right)^{-1}, \tag{20}$$

and

$$E, F = \frac{D}{2}\left(-1 \mp i\frac{1+p}{q}\right). \quad (21)$$

Finally, $t(y)$ is given by

$$t(y) = \frac{D}{-b}\Bigg\{ \ln\left(\frac{y-2p}{-4p}\sqrt{\frac{p^2+q^2}{(y+p)^2+q^2}}\right) + \frac{2+p}{2q} \times \left(\arctan\left(\frac{q}{y+p}\right) + \arctan\left(\frac{q}{p}\right) + \pi u(y+p)\right)\Bigg\}, \quad (22)$$

where $u(l) = 0$ while $l < 0$ otherwise 1 and where $-kA < y < kA$. If variables D, p and q are represented by a function of ak and kA, and if b is written as $f_b(ak, Ak)/k$, then, (22) becomes $k \times f(ak, Ak)$.

7. References

[1] Kocaoglu, M., Gulbahar, B., Akan, O.: 'Stochastic resonance in graphene bilayer optical nanoreceivers,' IEEE Trans. Nanotechnol., Nov 2014, 13, (6), pp. 1107–1117

[2] Liu, Z., Lai, Y.-C., Nachman, A.: 'Enhancement of noisy signals by stochastic resonance,' Physics Letters A, 2002, 297, (12), pp. 75 – 80

[3] Albert, T., Bulsara, A., Schmera, G., Inchiosa, M.: 'An evaluation of the stochastic resonance phenomenon as a potential tool for signal processing,' Conference Record of The Twenty-Seventh Asilomar Conference on Signals, Systems and Computers, Nov 1993, pp. 583–587 vol.1

[4] Benzi, R., Sutera, A., Vulpiani, A.: 'The mechanism of stochastic resonance,' Journal of Physics A: Mathematical and General, 1981, 14, (11), p. L453

[5] Benzi, R., Parisi, G., Sutera, A., Vulpiani, A.: 'Stochastic resonance in climatic change,' Tellus, 1982, 34, (1), pp. 10–16

[6] Gammaitoni, L., Hänggi, P., Jung, P., Marchesoni, F.: 'Stochastic resonance: A remarkable idea that changed our perception of noise,' European Physical Journal B, 2009, 69, (1), pp. 1–3

[7] Moss, F., Ward, L. M., Sannita, W. G.: 'Stochastic resonance and sensory information processing: a tutorial and review of application,' Clinical Neurophysiology, 2004, 115, (2), pp. 267 – 281

[8] McDonnell, M. D., Abbott, D.: 'What is stochastic resonance? definitions, misconceptions, debates, and its relevance to biology,' PLoS Comput Biol, 05 2009, 5, (5), p. e1000348

[9] Chen, H., Varshney, P., Kay, S., Michels, J.: ‘Theory of the stochastic resonance effect in signal detection: Part i mdash;fixed detectors,’ IEEE Trans. Signal Process., July 2007, 55, (7), pp. 3172–3184

[10] Chen, H., Varshney, P.: ‘Theory of the stochastic resonance effect in signal detection part ii: Variable detectors,’ IEEE Trans. Signal Process., Oct 2008, 56, (10), pp. 5031–5041

[11] McNamara, B., Wiesenfeld, K.: ‘Theory of stochastic resonance,’ Phys. Rev. A, May 1989, 39, (9), pp. 4854–4869

[12] Barbini, L., Cole, M. O. T., Hillis, A. J., Bois, du J. L.: ‘Weak signal detection based on two dimensional stochastic resonance,’ Signal Processing Conference (EUSIPCO), 2015 23rd European, Aug 2015, pp. 2147–2151

[13] Dubkov, A., Spagnolo, B., Valenti, D.: ‘New analytical approach to analyze the nonlinear regime of stochastic resonance,’ Noise and Fluctuations (ICNF), 2015 International Conference on, June 2015, pp. 1–4

[14] Zhang, X., Xu, W.: ‘Stochastic resonance in an asymmetric bistable system with coloured noises and periodic rectangular signal,’ Physica A: Statistical Mechanics and its Applications, 2007, 385, (1), pp. 95 – 104

[15] Guo, Y., Tan, J.: ‘Suprathreshold stochastic resonance in multilevel threshold system driven by multiplicative and additive noises,’ Communications in Nonlinear Science and Numerical Simulation, 2013, 18, (10), pp. 2852 – 2858

[16] Shao, R. H., Chen, Y.: ‘Stochastic resonance in time-delayed bistable systems driven by weak periodic signal,’ Physica A: Statistical Mechanics and its Applications, 2009, 388, (6), pp. 977 – 983

[17] Zozor, S., Amblard, P.-O.: ‘Stochastic resonance in locally optimal detectors,’ IEEE Trans. Signal Process., Dec 2003, 51, (12), pp. 3177–3181

[18] Liu, J., Li, Z., Guan, L., Pan, L.: ‘A novel parameter-tuned stochastic resonator for binary pam signal processing at low snr,’ IEEE Commun. Lett., March 2014, 18, (3), pp. 427–430

[19] Liu, J., Li, Z.: ‘Lowering the signal-to-noise ratio wall for energy detection using parameter-induced stochastic resonator,’ Communications, IET, 2015, 9, (1), pp. 101–107

[20] Mitaim, S., Kosko, B.: ‘Adaptive stochastic resonance,’ Proc. IEEE, Nov 1998, 86, (11), pp. 2152–2183

[21] Gammaitoni, L., Hänggi, P., Jung, P., Marchesoni, F.: ‘Stochastic resonance,’ Reviews of Modern Physics, 1998, 70, (1), pp. 223–287

[22] Moss, F.: ‘Stochastic resonance: a signal+noise in a two state system,’ Proceedings of the 45th Annual Symposium on Frequency Control, May 1991, pp. 649–658

[23] Misono, M., Kohmoto, T., Fukuda, Y., Kunitomo, M.: ‘Stochastic resonance in an optical bistable system driven by colored noise,’ Optics Communications, 1998, 152, (4-6), pp. 255 – 258

[24] Ai-Jie, L., Lian-Cun, Z., Lian-Xi, M., Xin-Xin, Z.: 'Analytical solutions of a model for brownian motion in the double well potential,' Communications in Theoretical Physics, 2015, 63, (1), p. 51

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