

## Probabilistic sensitivity analysis of suspension bridges to near-fault ground motion

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**Abstract.** The sensitivities of a structural response due to variation of its design parameters are prerequisite in the majority of the algorithms used for fundamental problems in engineering as system uncertainties, identification and probabilistic assessments etc. The paper presents the concept of probabilistic sensitivity of suspension bridges with respect to near-fault ground motion. In near field earthquake ground motions, large amplitude spectral accelerations can occur at long periods where many suspension bridges have significant structural response modes. Two different types of suspension bridges, which are Bosphorus and Humber bridges, are selected to investigate the near-fault ground motion effects on suspension bridges random response sensitivity analysis. The modulus of elasticity is selected as random design variable. Strong ground motion records of Kocaeli, Northridge and Erzincan earthquakes are selected for the analyses. The stochastic sensitivity displacements and internal forces are determined by using the stochastic sensitivity finite element method and Monte Carlo simulation method. The stochastic sensitivity displacements and responses obtained from the two different suspension bridges subjected to these near-fault strong-ground motions are compared with each other. It is seen from the results that near-fault ground motions have different impacts stochastic sensitivity responses of suspension bridges. The stochastic sensitivity information provides a deeper insight into the structural design and it can be used as a basis for decision-making.

**Keywords:** probabilistic sensitivity; stochastic sensitivity finite element method; Monte Carlo Simulation; suspension bridge

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### 1. Introduction

The sensitivities of a structural response due to variation of its design parameters are prerequisite in the majority of the algorithms used for fundamental problems in engineering. In structural sensitivity analysis, deterministic procedure is insufficient to provide complete information. Design sensitivity analysis takes into account variations of the design parameters and develops appropriate procedures for calculating a change of structural behavior that depends implicitly on the design variables. The theory of structural sensitivity in deterministic description has become an advanced branch of the present-day structural engineering (Chen *et al.* 1992, Dutta *et al.* 1998, Kolakowski *et al.* 1998, Salary *et al.* 1993). Conventional sensitivity analysis is

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deterministic in nature and neglects the effect of inherent randomness associated with the input parameters. But the reality claims of unavoidable inflow of uncertainty in distribution of design variables. Thus, deterministic sensitivity analysis is insufficient to provide complete information regarding structural response.

The concept of stochastic sensitivity, aims to find the expectations and the sensitivity response for changes in the structural response due to structural parameter variations. The sensitivity analysis of any structural system involves computation of the derivatives of the structural response quantities like displacements, strains, stresses, eigenvalues, eigenvectors etc. These uncertainties can be illustrated geometrical characteristics (cross-sectional area, flexural inertia, length etc.), material characteristics (young's modulus, poisson's ratio etc.), initial connection stiffness and magnitudes and distributions of loads etc. and affect sensitivity gradient. Stochastic finite element method (SFEM) is a holistic approach for computing response sensitivity considering uncertainties in structural parameters. The SFEM can be well explored to assess the sensitivity of structural systems due to the random variation of design parameters (Kleiber and Hien 1992). In fact, in probabilistic structural analysis, such as the mean-based second-moment method and reliability-based optimization, stochastic sensitivities provide important information for updating the mean values of the random design parameters. Hence, it is of considerable practical importance to estimate the effect of uncertainty in structural parameters over response sensitivity. The SFEM approach is numerically much more efficient than statistical techniques such as the Monte Carlo simulation, since only the first two moments need to be given on input. The formulation was based on the transformation of the set of correlated random variables into a set of uncorrelated random variables through a standard eigen problem (Vanmarcke *et al.* 1986, Schueller 1997, Vanmarcke *et al.* 1999, Çavdar *et al.* 2010). Though the literature on the stochastic sensitivity analysis is limited, attempts have been made to exploit the perturbation technique for the sensitivity analysis of uncertain structures for static and dynamic problems (Hien *et al.* 1991, Juhn *et al.* 1992, Lee *et al.* 1997, Song *et al.* 1995, Chaudhuri *et al.* 1995, Chao *et al.* 2011).

Ground motions in near source region of large crustal earthquakes are significantly affected by rupture directivity and tectonic fling. These effects are the strongest at longer periods and they can have a significant impact on Engineering Structures. Near-fault (NF) ground motions are ground motions that occur near an earthquake fault. NF motions recorded near the epicenter are known as near-epicenter records, while those obtained along the fault in the direction of rupture are referred to as being in the region of forward directivity. NF motions in the region of forward directivity may cause a large peak horizontal elastic response in structures. Comparison of the near-fault strong ground motions with far-fault strong ground motions is shown in Fig. 1. These pulses are strongly influenced by the orientation of the fault, the direction of slip on the fault and the location of the recording station relative to the fault, which is termed as 'directivity effect' due to the propagation of the rupture toward the recording site (Bray *et al.* 1995, Megawati *et al.* 2001, Wang *et al.* 2002, Bayraktar *et al.* 2009).

The effects of NF ground motion on many civil engineering structures such as buildings, tunnels, bridges, nuclear station, etc., have been investigated in many recent studies (Malhotra *et al.* 1999, Liao *et al.* 2001, Liao *et al.* 2004, Dicleli *et al.* 2007). It can be clearly seen from these studies that the importance of near-fault ground motion effect on the response of the structures have been highlighted. Dynamic responses of bridges have been investigated using SFEM by many researches (Vanmarcke *et al.* 1986, Kleiber *et al.* 1992, Zhu *et al.* 1992, Zhang *et al.* 1996, Çavdar *et al.* 2010). The stochastic sensitivity finite element method (SSFEM) for structures has been developed by several researchers (Hien *et al.* 1991, Juhn *et al.* 1992, Lee *et al.* 1997, Song *et*

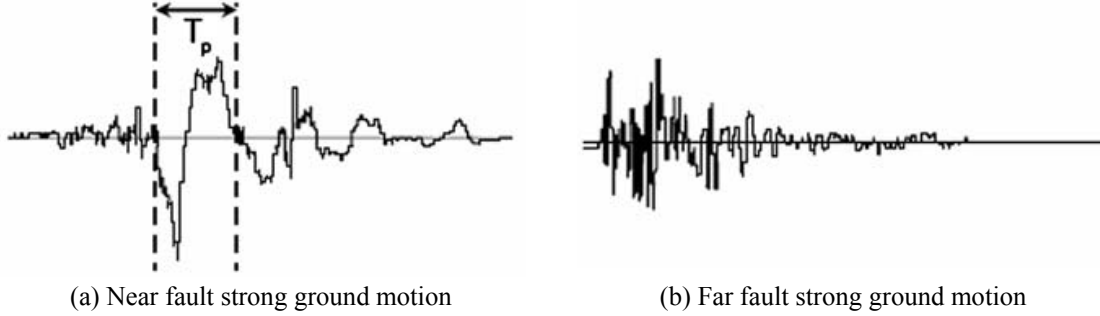


Fig. 1 The time-histories of two different strong ground motion velocities

*al.* 1995, Bhattacharyya *et al.* 2002). However, most of their work is limited to simple structures. There is no sufficient research about the near-fault ground motion effect on the stochastic sensitivity analysis of complex suspension bridges.

The main objective of this paper is to present an algorithm for evaluating the analytical stochastic sensitivity of dynamic responses of long span suspension bridges subjected to NF earthquake load. The proposed formulation computes the analytical stochastic sensitivity gradient with respect to the random structural design parameters. For this purpose, two different types of suspension bridges, which are Bosphorus and Humber bridges, are selected to investigate the near-fault ground motion effects on suspension bridges random response sensitivity analysis. As the conventional deterministic sensitivity analysis cannot provide complete information, stochastic sensitivity analysis is needed to tackle the uncertainties in structural parameters. The stochastic sensitivity behavior of suspension bridges is selected modulus of elasticity as random design variable. Strong ground motion records of Kocaeli (1999), Northridge (1994) and Erzincan (1992) earthquakes are selected for the analyses. In this study, the term “near-fault ground motion” is referred to the ground motion record obtained in the vicinity of a fault with apparent velocity pulse (pulse duration larger than 1.0 s), and the peak ground velocity/peak ground acceleration (PGV/PGA) value (unit is second) which is larger than 0.1 s. All of the records were obtained from sites with epicentral distances of less than 10 km.

## 2. The Stochastic Sensitivity Finite Element Method (SSFEM)

The concept of stochastic sensitivity aims to find the expectations and sensitivity response for changes in the structural response due to structural parameter variations. Structural response sensitivity of multi-degree-of-freedom dynamic systems is considered. Both the time interval and time instant response sensitivities are considered here in the context of stochastic behavior. This paper deals with random displacement sensitivity when the structure involves the modulus of elasticity ( $E$ ) as a random parameter.

A deterministic equation of motion can be written as

$$M_{\alpha\beta}\ddot{q}_{\beta} + C_{\alpha\beta}\dot{q}_{\beta} + K_{\alpha\beta}q_{\beta} = Q_{\alpha} \quad (1)$$

where  $K_{\alpha\beta}$ ,  $M_{\alpha\beta}$ ,  $C_{\alpha\beta}$  denote the stiffness matrix, mass matrix and damping matrix,  $\ddot{q}_{\beta}$ ,  $\dot{q}_{\beta}$ ,  $q_{\beta}$  denote

the acceleration, velocity, displacement, respectively. The stochastic perturbation based approach consists usually of the up to the second order equations obtained starting from the deterministic ones.

The objective of the stochastic sensitivity analysis is to determine changes in the structural response functional with variations in design parameters.

For a linear elastic system with  $N$  degrees of freedom, consider the system response over the time interval  $[0, T]$  described by the integral functional (Kleiber *et al.* 1992, Kleiber *et al.* 1997).

$$\begin{aligned} \varphi(h^d, b_\rho) &= \int_0^T G[q_\alpha(h^d, b_\rho; \tau), h^d] d\tau \\ d &= 1, 2, \dots, D; \rho = 1, 2, \dots, \tilde{N}; \alpha = 1, 2, \dots, N \end{aligned} \quad (2)$$

where  $G$  is a given function of its arguments,  $h^d$  is a  $D$  dimensional design variable vector,  $b_\rho$  is an  $\tilde{N}$  – dimensional random variable vector and  $q_\alpha$  is an  $N$ -dimensional vector of nodal displacement-type variables.

The nodal displacement-type variables  $q_\alpha(h^d, b_\rho; \tau)$  are implicit functions of the random and design variables and satisfy the spatially discretized equation of motion of the form.

Zeroth-order equation ( $\epsilon^0$  terms, one pair of systems of  $N$  linear simultaneous ordinary differential equations for  $q_\alpha^0(h^d, b_\rho^0; \tau)$  and  $\lambda_\alpha^0(h^d, b_\rho^0; \tau)$ ,  $\tau \in [0, T]$ , respectively)

$$M_{\alpha\beta}^0(h^d, b_\rho^0) \ddot{q}_\beta^0(h^d, b_\rho^0; \tau) + C_{\alpha\beta}^0(h^d, b_\rho^0) \dot{q}_\beta^0(h^d, b_\rho^0; \tau) + K_{\alpha\beta}^0(h^d, b_\rho^0) q_\beta^0(h^d, b_\rho^0; \tau) = Q_\alpha^0(h^d, b_\rho^0; \tau)$$

together with the homogeneous initial conditions

$$\begin{aligned} q_\alpha^0(h^d, b_\rho^0; 0); \dot{q}_\alpha^0(h^d, b_\rho^0; 0) &= 0 \\ M_{\alpha\beta}^0(h^d, b_\rho^0) \ddot{\lambda}_\beta^0(h^d, b_\rho^0; \tau) - C_{\alpha\beta}^0(h^d, b_\rho^0) \dot{\lambda}_\beta^0(h^d, b_\rho^0; \tau) \\ + K_{\alpha\beta}^0(h^d, b_\rho^0) \lambda_\beta^0(h^d, b_\rho^0; \tau) &= G_\alpha^0(h^d, b_\rho^0; \tau) \\ (h^d, b_\rho; T) &= 0; \dot{\lambda}_\alpha^0(h^d, b_\rho; T) = 0 \end{aligned} \quad (3)$$

First-order equations, ( $\epsilon^1$  terms,  $\tilde{N}$  pairs systems of  $N$  linear simultaneous ordinary differential equations for

$$\begin{aligned} q_\alpha^{\rho} (h^d, b_\rho^0; \tau) \text{ and } \lambda_\alpha^{\rho} (h^d, b_\rho^0; 0), \tau \in [0, T], \text{ respectively} \\ M_{\alpha\beta}^0(h^d, b_\rho^0) \ddot{q}_\beta^{\rho}(h^d, b_\rho^0; \tau) + C_{\alpha\beta}^0(h^d, b_\rho^0) \dot{q}_\beta^{\rho}(h^d, b_\rho^0; \tau) \\ + K_{\alpha\beta}^0(h^d, b_\rho^0) q_\beta^{\rho}(h^d, b_\rho^0; \tau) &= Q_\alpha^{\rho}(h^d, b_\rho^0; \tau) \\ (h^d, b_\rho; 0) &= 0; \dot{q}_\alpha^{\rho}(h^d, b_\rho; 0) = 0 \\ M_{\alpha\beta}^0(h^d, b_\rho^0) \ddot{\lambda}_\beta^{\rho}(h^d, b_\rho^0; \tau) - C_{\alpha\beta}^0(h^d, b_\rho^0) \dot{\lambda}_\beta^{\rho}(h^d, b_\rho^0; \tau) \\ + K_{\alpha\beta}^0(h^d, b_\rho^0) \lambda_\beta^{\rho}(h^d, b_\rho^0; \tau) &= G_\alpha^{\rho}(h^d, b_\rho^0; \tau) \\ (h^d, b_\rho; T) &= 0; \dot{\lambda}_\alpha^{\rho}(h^d, b_\rho; T) = 0, \rho = 1, 2, \dots, \tilde{N} \end{aligned} \quad (4)$$

Second-order ( $\epsilon^2$  terms, one pair of systems of  $N$  linear simultaneous ordinary differential equations for

$$\begin{aligned}
 & q_\alpha^{(2)}(h^d, b_\rho^0; \tau) \text{ and } \dot{\lambda}_\alpha^{(2)}(h^d, b_\rho^0; \tau), \tau \in [0, T], \text{ respectively} \\
 & M_{\alpha\beta}^0(h^d, b_\rho^0) \ddot{q}_\beta^{(2)}(h^d, b_\rho^0; \tau) + C_{\alpha\beta}^0(h^d, b_\rho^0) \dot{q}_\beta^{(2)}(h^d, b_\rho^0; \tau) \\
 & \quad + K_{\alpha\beta}^0(h^d, b_\rho^0) q_\beta^{(2)}(h^d, b_\rho^0; \tau) = Q_\alpha^{(2)}(h^d, b_\rho^0; \tau) \\
 & \quad q_\alpha^{(2)}(h^d, b_\rho^0; 0) = 0; \quad \dot{q}_\alpha^{(2)}(h^d, b_\rho^0; 0) = 0 \\
 & M_{\alpha\beta}^0(h^d, b_\rho^0) \ddot{\lambda}_\beta^{(2)}(h^d, b_\rho^0; \tau) - C_{\alpha\beta}^0(h^d, b_\rho^0) \dot{\lambda}_\beta^{(2)}(h^d, b_\rho^0; \tau) \\
 & \quad + K_{\alpha\beta}^0(h^d, b_\rho^0) \lambda_\beta^{(2)}(h^d, b_\rho^0; \tau) = G_\alpha^{(2)}(h^d, b_\rho^0; \tau) \\
 & \quad \lambda_\alpha^{(2)}(h^d, b_\rho^0; T) = 0; \quad \dot{\lambda}_\alpha^{(2)}(h^d, b_\rho^0; T) = 0
 \end{aligned} \tag{5}$$

In Eqs. (3)-(5) the indices run over the following sequence

$$\begin{aligned}
 q_\alpha^{(2)}(h^d, b_\rho^0; \tau) &= q_\alpha^{\cdot\rho\sigma}(h^d, b_\rho^0; \tau) S_b^{\rho\sigma} \\
 \lambda_\alpha^{(2)}(h^d, b_\rho^0; \tau) &= \lambda_\alpha^{\cdot\rho\sigma}(h^d, b_\rho^0; \tau) S_b^{\rho\sigma} \quad \rho, \sigma = 1, 2, \dots, \tilde{N}
 \end{aligned} \tag{6}$$

In Eqs. (4)-(5) the first and second order primary and adjoint generalized load vectors are denoted by

$$Q_\alpha^\rho(h^d, b_\ell^0; \tau) = Q_\alpha^{\cdot\rho}(h^d, b_\ell^0; \tau) - \left[ \begin{array}{l} M_{\alpha\beta}^{\cdot\rho}(h^d, b_\ell^0) \ddot{q}_\beta^0(h^d, b_\ell^0; \tau) \\ + C_{\alpha\beta}^{\cdot\rho}(h^d, b_\ell^0) \dot{q}_\beta^0(h^d, b_\ell^0; \tau) + K_{\alpha\beta}^{\cdot\rho}(h^d, b_\ell^0) q_\beta^0(h^d, b_\ell^0; \tau) \end{array} \right] \tag{7}$$

$$G_\alpha^\rho(h^d, b_\ell^0; \tau) = G_\alpha^{\cdot\rho}(h^d, b_\ell^0; \tau) - \left[ \begin{array}{l} M_{\alpha\beta}^{\cdot\rho}(h^d, b_\ell^0) \dot{\lambda}_\beta^0(h^d, b_\ell^0; \tau) \\ + C_{\alpha\beta}^{\cdot\rho}(h^d, b_\ell^0) \lambda_\beta^0(h^d, b_\ell^0; \tau) + K_{\alpha\beta}^{\cdot\rho}(h^d, b_\ell^0) \lambda_\beta^0(h^d, b_\ell^0; \tau) \end{array} \right]$$

$$\begin{aligned}
 Q_\alpha^{(2)}(h^d, b_\ell^0; \tau) &= \left\{ \begin{array}{l} Q_\alpha^{\cdot\rho\sigma}(h^d, b_\ell^0; \tau) - 2 \left[ \begin{array}{l} M_{\alpha\beta}^{\cdot\rho}(h^d, b_\ell^0) \ddot{q}_\beta^{\cdot\sigma}(h^d, b_\ell^0; \tau) \\ + C_{\alpha\beta}^{\cdot\rho}(h^d, b_\ell^0) \dot{q}_\beta^{\cdot\sigma}(h^d, b_\ell^0; \tau) + K_{\alpha\beta}^{\cdot\rho}(h^d, b_\ell^0) q_\beta^{\cdot\sigma}(h^d, b_\ell^0; \tau) \end{array} \right] \\ - \left[ \begin{array}{l} M_{\alpha\beta}^{\cdot\rho\sigma}(h^d, b_\ell^0) \ddot{q}_\beta^0(h^d, b_\ell^0; \tau) + C_{\alpha\beta}^{\cdot\rho\sigma}(h^d, b_\ell^0) \dot{q}_\beta^0(h^d, b_\ell^0; \tau) \\ + K_{\alpha\beta}^{\cdot\rho\sigma}(h^d, b_\ell^0) q_\beta^0(h^d, b_\ell^0; \tau) \end{array} \right] \end{array} \right\} S_b^{\rho\sigma} \\
 G_\alpha^{(2)}(h^d, b_\ell^0; \tau) &= \left\{ \begin{array}{l} G_\alpha^{\cdot\rho\sigma}(h^d, b_\ell^0; \tau) - 2 \left[ \begin{array}{l} M_{\alpha\beta}^{\cdot\rho}(h^d, b_\ell^0) \dot{\lambda}_\beta^{\cdot\sigma}(h^d, b_\ell^0; \tau) \\ + C_{\alpha\beta}^{\cdot\rho}(h^d, b_\ell^0) \lambda_\beta^{\cdot\sigma}(h^d, b_\ell^0; \tau) + K_{\alpha\beta}^{\cdot\rho}(h^d, b_\ell^0) \lambda_\beta^{\cdot\sigma}(h^d, b_\ell^0; \tau) \end{array} \right] \\ - \left[ \begin{array}{l} M_{\alpha\beta}^{\cdot\rho\sigma}(h^d, b_\ell^0) \dot{\lambda}_\beta^0(h^d, b_\ell^0; \tau) + C_{\alpha\beta}^{\cdot\rho\sigma}(h^d, b_\ell^0) \lambda_\beta^0(h^d, b_\ell^0; \tau) \\ + K_{\alpha\beta}^{\cdot\rho\sigma}(h^d, b_\ell^0) \lambda_\beta^0(h^d, b_\ell^0; \tau) \end{array} \right] \end{array} \right\} S_b^{\rho\sigma}
 \end{aligned} \tag{8}$$

where  $\tau$  is forward time variable,  $\tilde{N}$  is the number of nodal random variables.  $M_{\alpha\beta}^0$ ,  $C_{\alpha\beta}^0$  and  $K_{\alpha\beta}^0$  are system mass matrix, damping matrix and system stiffness matrix, respectively.  $Q_{\alpha}^0$ ,  $q_{\beta}^0$  and  $N$  are load vector, displacement of the nodal random variable and the number of degrees of freedom in the system respectively.  $S_b^{\rho\sigma}$  is covariance matrix of nodal random variables.  $(\cdot)^0$  is zeroth-order quantities, taken at means of random variables,  $(\cdot)^{\rho}$  is first partial derivatives with respect to nodal random variables,  $(\cdot)^{\rho\sigma}$  is second partial derivatives with respect to nodal random variables.

A direct Monte Carlo simulation (MCS) is also performed for comparison of results. The MCS method is a quite versatile mathematical tool capable of handling situations where all other methods fail to succeed; in structural dynamics, it has attracted intense attention only recently following the widespread availability of inexpensive computational systems (Shinozuka *et al.* 1972). A sample global stiffness matrix is formed on the basis of stochastic fields generated by means of the covariance matrix decomposition algorithm. The response sensitivity of the structure is determined by relying on the standard deterministic sensitivity analysis. This procedure is repeated several times to procedure an ensemble of the structural response sensitivity. In MCS, the random stiffness matrix needs to be inverted for each simulated structure consuming an enormous amount of CPU time.

### 3. Near-fault ground motions

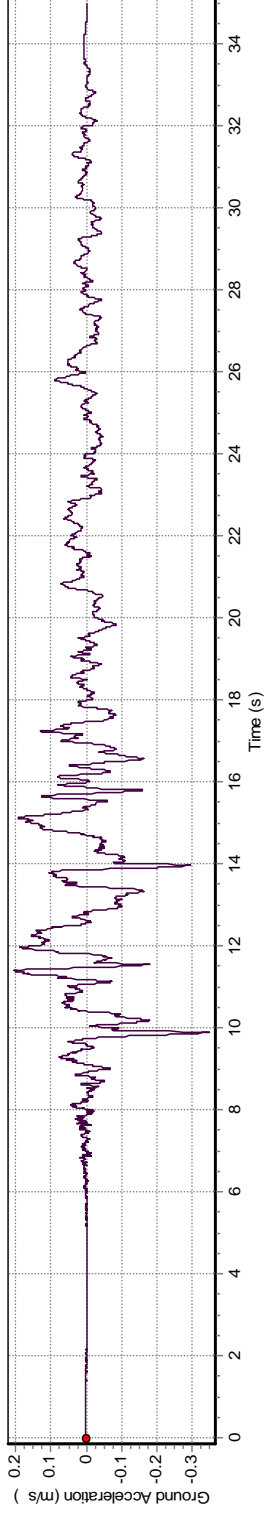
The effects of near-fault ground motion on the stochastic sensitivity seismic performance of Bosphorus and Humber suspension bridges should be investigated because of four significant faults. The strong ground motions of the Kocaeli (1999), Erzincan (1992) and Northridge (1994) earthquakes recorded near-fault are considered in the analysis.

The KOCAELI/YPT330 component of Yarimca station recorded during the Kocaeli earthquake in 1999, the ERZIKAN/ERZ-NS component of 95 Erzincan station recorded during the Erzincan earthquake in 1992, the NORTHRI/JEN022 component of 0655 Jensen Filter Plant station recorded during the Northridge earthquake in 1994 are used as ground motions. PGA and PGV, surface projection distances from the site to the fault and PGV/PGA values are depicted in Table 1. When selecting the near-fault ground motions it is considered that the ground motions have similar properties such as near peak acceleration value to compare the effects different near-fault ground motion on suspension bridges stochastic sensitivity response. The time histories for acceleration and velocity of records are presented in Figs. 2 and 3 respectively. The ground motion records are obtained from the PEER Strong Motion Database (PEER 2012).

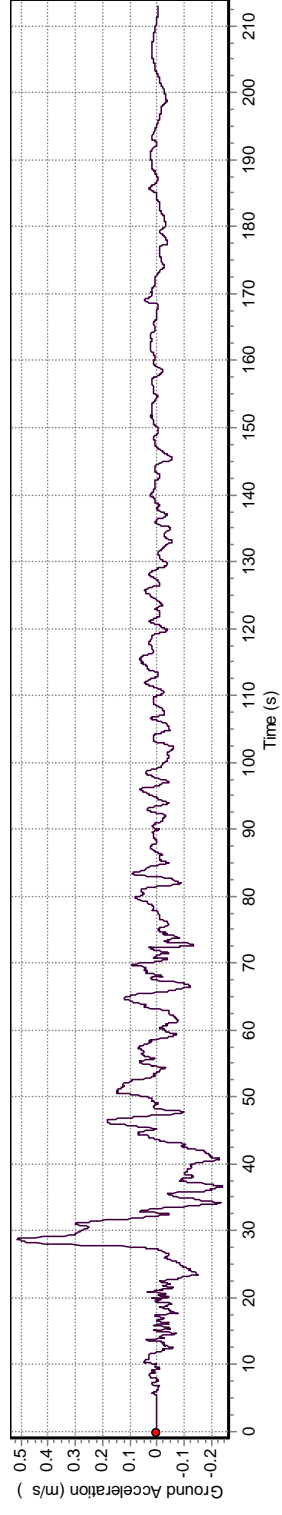
The Kocaeli, Turkey (1999), earthquake was recorded during the 7.4 magnitude earthquake; that magnitude was the highest considered in this study. The distance of the recording site from the source ranged from 2.0-6.2 km. A scatter plot of magnitude-distance pairs for the strong ground motion records is shown in Fig. 4. All of the records were obtained from sites with epicentral distances of less than 10 km.

Table 1 Properties of selected near-fault ground motion records

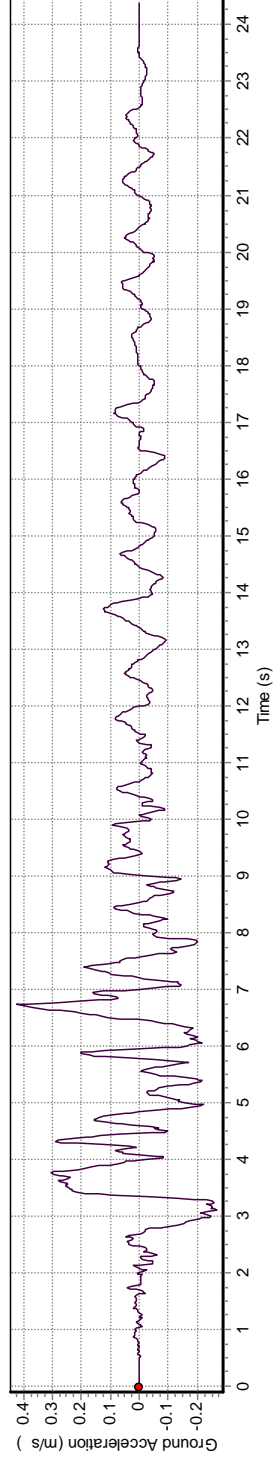
No	Earthquake	Station	PGA (m/s <sup>2</sup> )	PGV (cm/sn)	PGV/PGA (s)	Mw	Distance to fault (km)
1	Kocaeli	YPT330	0.349g	62.1	0.181	7.4	2.6
2	Erzincan	ERZ-NS	0.515g	83.9	0.166	6.9	2.0
3	Northridge	JEN022	0.424g	106.2	0.255	6.7	6.2



(a) The time-histories of ground motion acceleration subjected to Kocaeli 1999

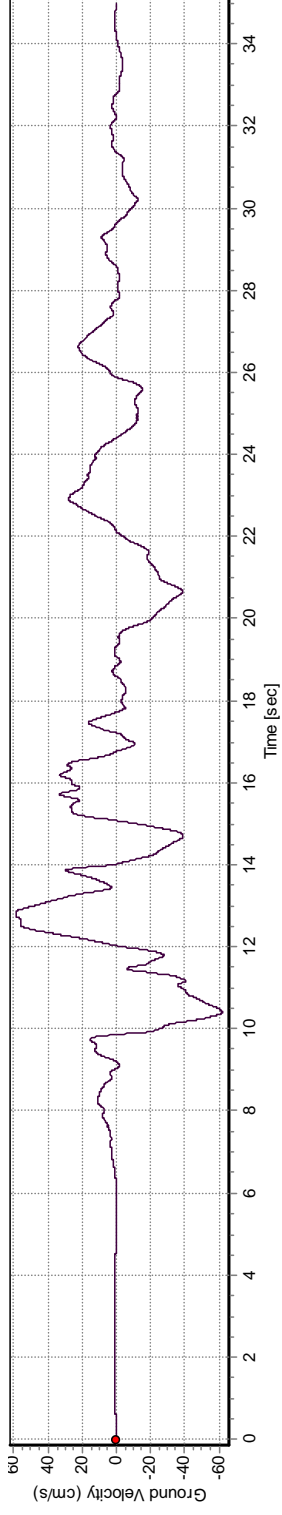


(b) The time-histories of ground motion acceleration subjected to Erzincan 1992

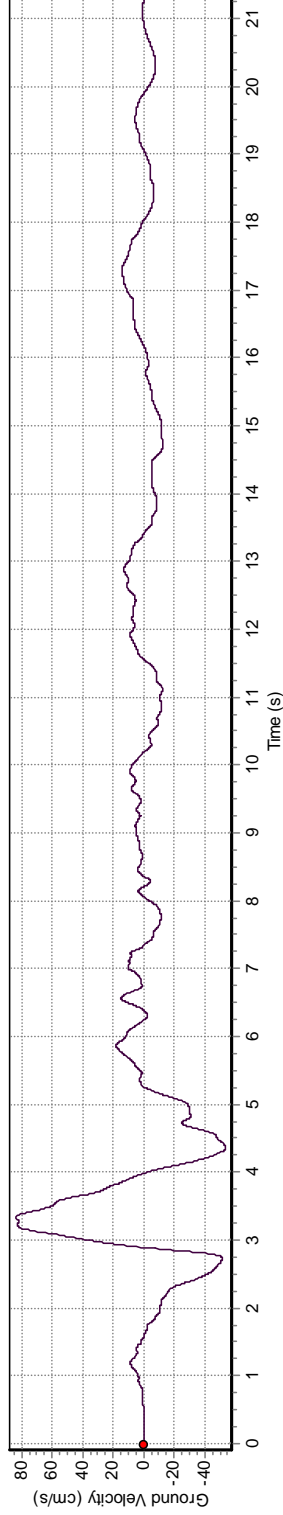


(c) The time-histories of ground motion acceleration subjected to Northridge 1994

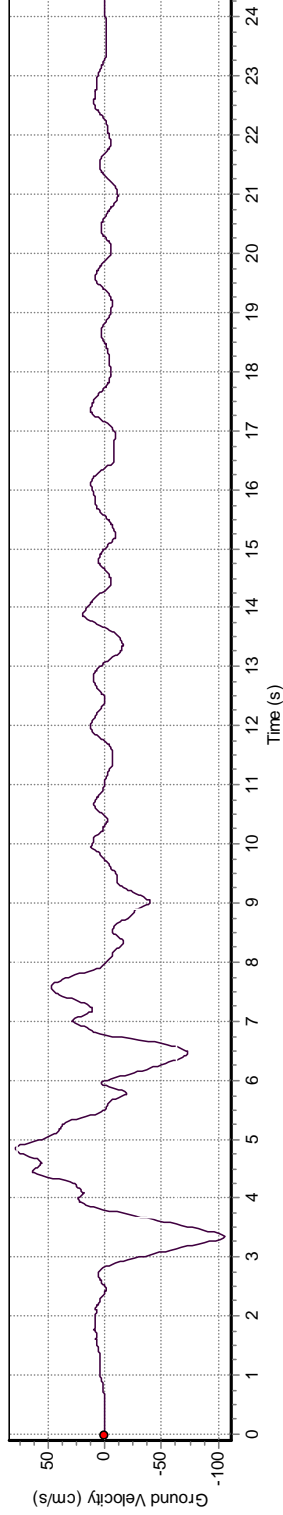
Fig. 2 The time-histories of near-fault ground motion acceleration for selected earthquakes



(a) The time-histories of ground motion velocities subjected to Kocaeli 1999



(b) The time-histories of ground motion velocities subjected to Erzincan 1992



(c) The time-histories of ground motion velocities subjected to Northridge 1994

Fig. 3 The time-histories of near-fault ground motion velocities for selected earthquakes



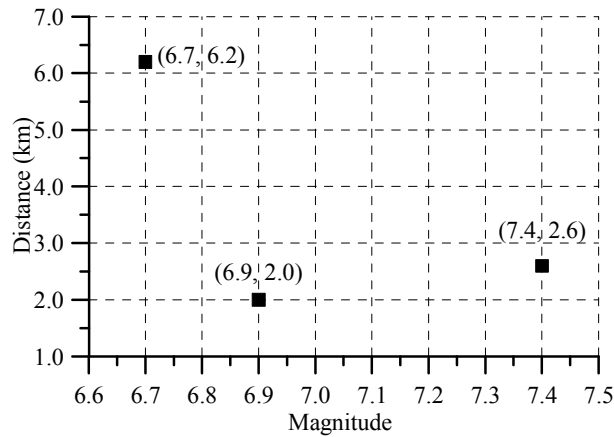


Fig. 4 Magnitude-distance distribution

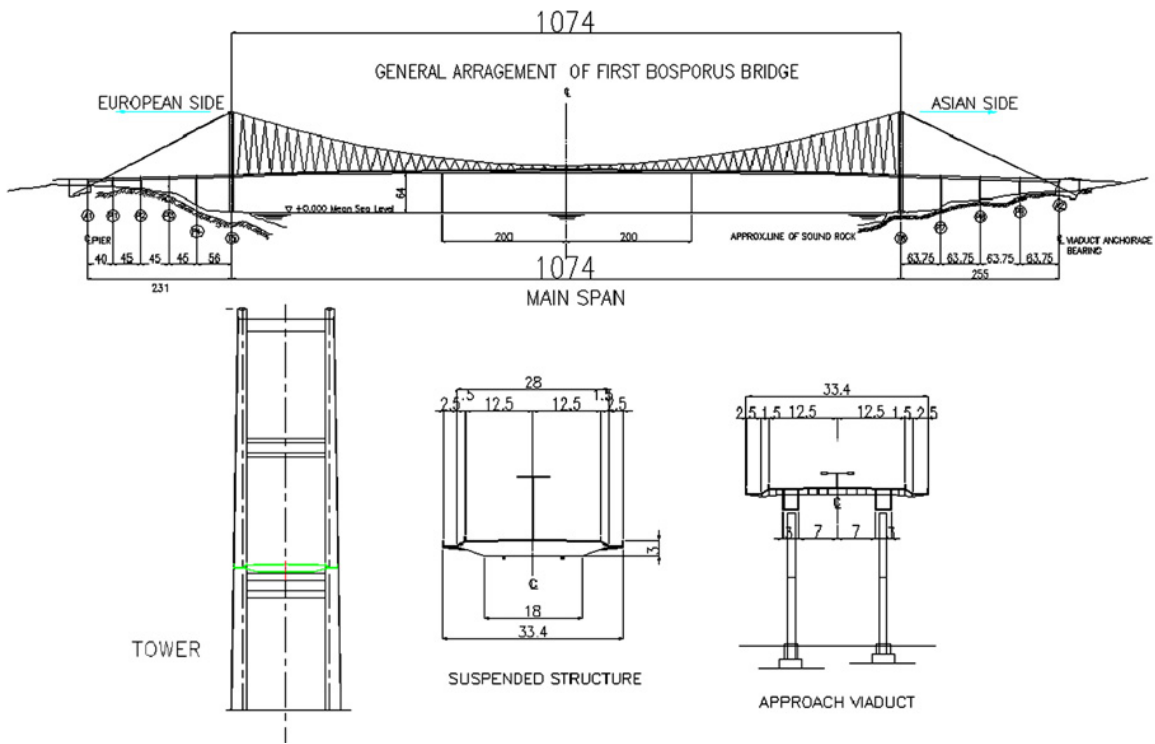


Fig. 5 General arrangement of Bosphorus Suspension Bridge

#### 4. Numerical examples

To illustrate the effectiveness of stochastic sensitivity analysis using the stochastic sensitivity finite element method (SSFEM) and Monte Carlo simulation (MCS) technique presented in the earlier sections on the near-fault (NF) ground motion, two example problems are presented.



Fig. 6 View of Bosphorus Suspension Bridge

#### 4.1 Example 1

Bosphorus suspension bridge, commissioned in 1973, joins the European and Asian Continents through Ortakoy and Beylerbeyi districts of Istanbul. It is a gravity-anchored suspension bridge with steel pylons and inclined hangers. The bridge has a main span of 1074 m (World rank 12th) and two side spans of 231 m and 255 m on the European and the Asian sides, respectively. The bridge has slender steel towers of 165 m high, a steel box-deck and inclined hangers. The horizontal distance between the cables is 28 m and the roadway is 21 m wide, accommodating three lanes each way. The roadway at the mid-span of the bridge is approximately 64 m above the sea level. The side span decks are not connected to the cable and the decks rest on piers taken to foundation level. The cost of the bridge amounted to USD 200 million (Adanur, 2003). General arrangement of the Bosphorus Suspension Bridge is shown in Fig. 5. In addition, material and sectional properties such as main cable, back stay cable, hangers, deck etc. of Bosphorus Suspension Bridge were shown in Table 2.

Table 2 Structural material and sectional properties of Bosphorus Suspension Bridge (Adanur 2003)

Members	Elastic modulus (kN/m <sup>2</sup> )	Cables sectional area (m <sup>2</sup> )	Moments of inertia (m <sup>4</sup> )	Poisson's ratio	Mass density (ton/m <sup>3</sup> )
Deck	$2.05 \times 10^8$	0.861	1.238	0.3	14.97
Hanger	$1.62 \times 10^8$	0.0021	$3.068 \times 10^{-7}$	0.3	8.004
Main cable	$1.93 \times 10^8$	0.205	$3.344 \times 10^{-3}$	0.3	8.629
Backstay cable	$1.93 \times 10^8$	0.219	$3.817 \times 10^{-3}$	0.3	8.334
Tower	$2.05 \times 10^8$	0.68	4.9	0.3	7.85

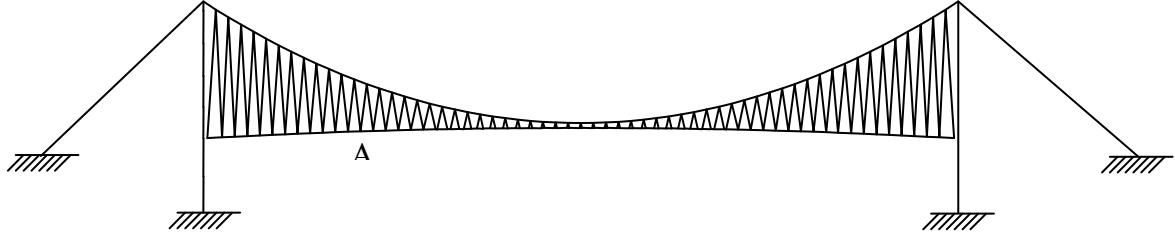


Fig. 7 Two-dimensional finite element model of Bosphorus Suspension Bridge

To investigate the stochastic sensitivity response of the Bosphorus Suspension Bridge under to near fault ground motion, two-dimensional mathematical model are used for calculations. Dumanoglu and Severn (1990) verified that 2D analysis provide natural frequencies and mode shapes which are in close agreement with those obtained by 3D analysis in the vertical direction for suspension bridges. The finite element model of Bosphorus Suspension Bridge is shown in Fig. 7. As the deck, towers and cables are represented by beam elements, the hangers are represented by truss elements in the model. Because the side span decks are not connected to the cable, they are not considered in the finite element models. Finite element model of the Bosphorus Suspension Bridge with inclined hangers has 161 nodal points, 159 beam elements and 118 truss elements and the model is represented by 469 degrees of freedom. This model has three degrees of freedom at each nodal point, namely, two translational degrees of freedom in vertical and longitudinal axes and one rotational degree of freedom in lateral axis.

The suspension bridge is modeled by 277 stochastic finite elements of different length. The respective expectation and correlation function for the elastic modulus  $E_p$  are assumed as follows

When the time-instant sensitivity response of the structural system of Fig. 7. The structural response functional is defined as

$$\varphi(\tau) = \frac{[q_\beta(\tau)]^2}{(q_\beta^{(A)})^2} - 1 \leq 0$$

where  $q_\beta(\tau)$  is vertical displacement at the apex A and  $q_\beta^{(A)}$  is an admissible displacement value. The elastic module is assumed to be random design variables.

Elastic module from material properties is chosen as random variable for steel frame system. The other variables are considered as deterministic. This random variable is assumed to follow a normal distribution with the coefficient of variation 0.10. The respective expectation and correlation function and coefficient of variation for the elastic modulus  $E_p$  are assumed as follows

$$\begin{aligned} E[E_p] &= 2.1 \times 10^8 \quad \lambda = 10 \\ \mu(E_\rho, E_\sigma) &= \exp\left(-\frac{|x_\rho - x_\sigma|}{\lambda l}\right) \quad \rho, \sigma = 1, 2, \dots, 277 \\ \alpha &= 0.10, \end{aligned} \quad (9)$$

where  $x_p$ ,  $l$  and  $\lambda$  are ordinates of the element midpoints ( $n$  random variable,  $\rho, \sigma = 1, 2, \dots, n$ ),

structural member length and decay factor, respectively. The Bosphorus suspension bridge is modeled by 277 stochastic finite elements with different lengths. MCS method is simulated for 10000 simulations.

Structural response sensitivity of multi-degree-of-freedom suspension bridges is considered for near-fault ground motion in this paper. The stochastic sensitivity of maximum displacements and internal forces of the suspension bridges are calculated according to Stochastic Sensitivity Finite Element Method (SSFEM) for near-fault ground motions. The efficiency and accuracy of the proposed algorithm are validated by comparison with results of Monte Carlo Simulation (MCS) method.

In the first part of this study, Bosphorus Bridge sensitive responses with respect to random elastic modulus according to SSFEM and MCS methods are determined compared with each other. The absolute maximum vertical displacement responses of the bridge deck and horizontal displacements along the Bosphorus European tower obtained from SSFEM and MCS methods for near-fault ground motion are presented in Fig. 8. Two sensitive analyses give very close results each other at the 1/3 length distance from end of deck and at the top point of tower where maximum displacements occurred. The maximum stochastic sensitivity vertical displacements at deck point for Kocaeli, Erzincan and Northridge near-fault (NF) ground motions occurred as 175.20 cm, 54.55 cm and 105.13 cm, respectively. The maximum horizontal displacements at tower for Kocaeli, Erzincan and Northridge NF ground motions occurred as 11.68 cm, 4.79 cm

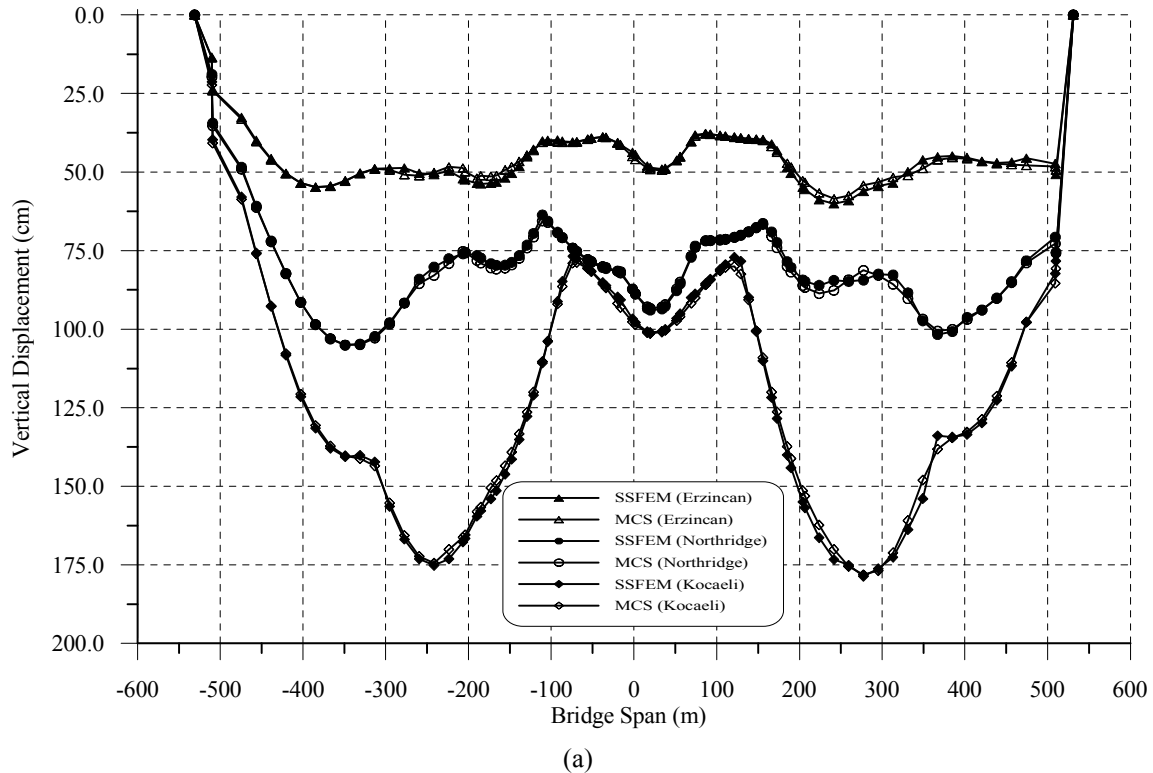


Fig. 8 Continued

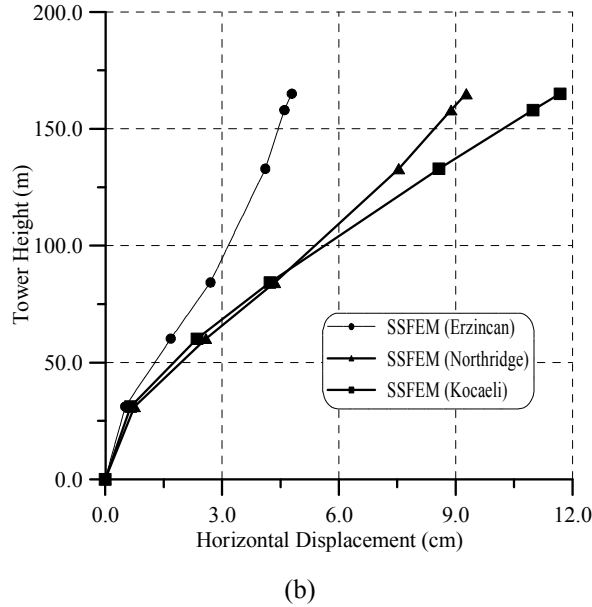


Fig. 8 Maximum sensitivity vertical displacements at the deck of Bosporus Bridge (a) and maximum horizontal displacements along Bosporus European tower (b) for random elastic modulus

and 9.27 cm, respectively. It is clearly seen that stochastic sensitive values of vertical and horizontal displacements obtained from Kocaeli earthquake ground motion are the highest.

By comparing SSFEM and MCS methods gives closer results to each other. The average

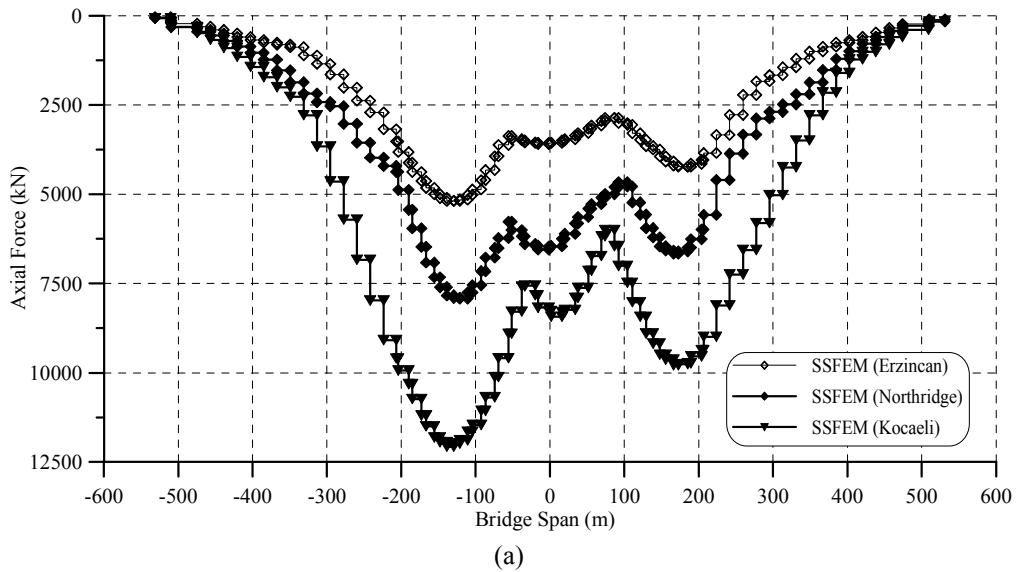


Fig. 9 Continued

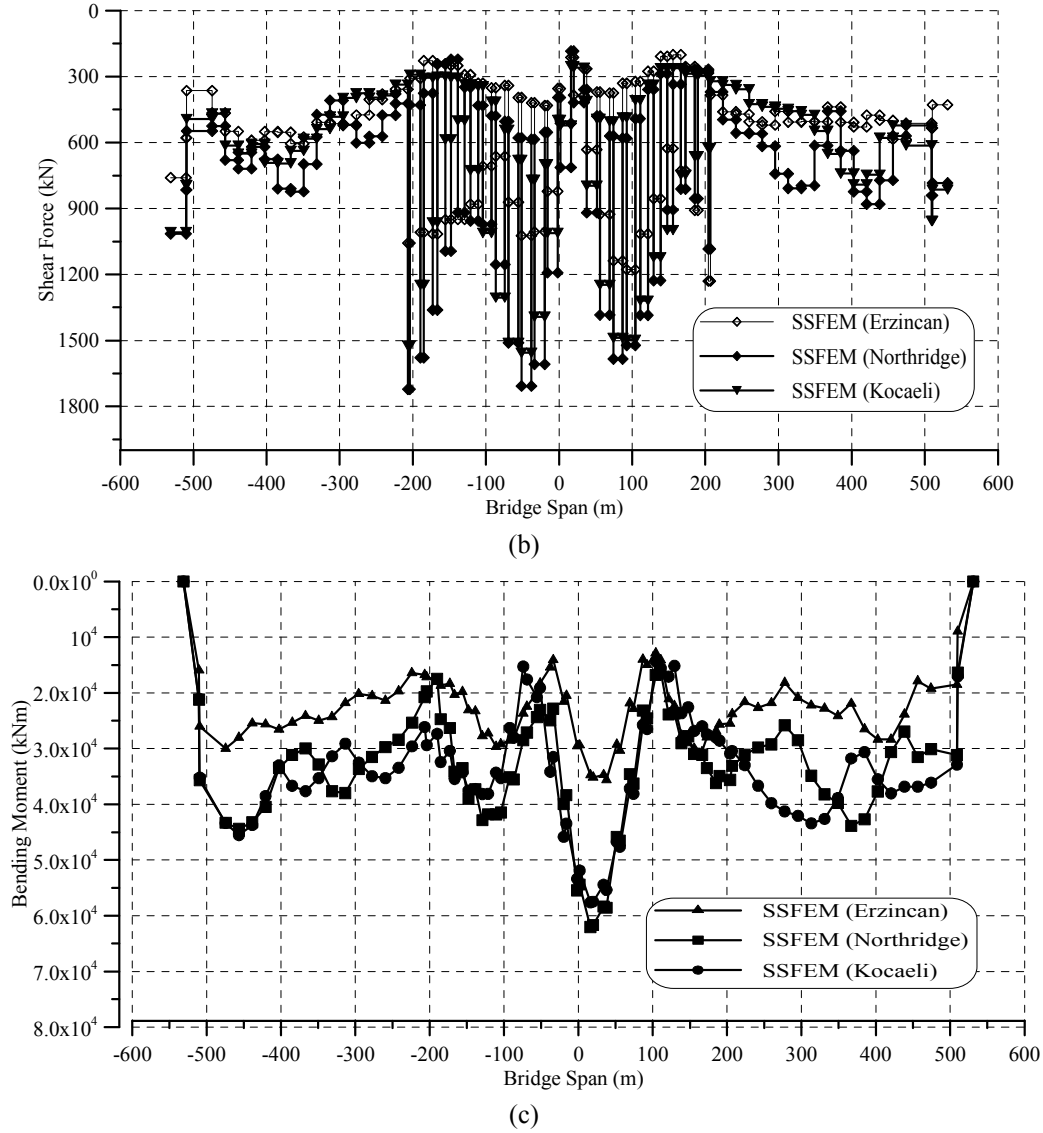


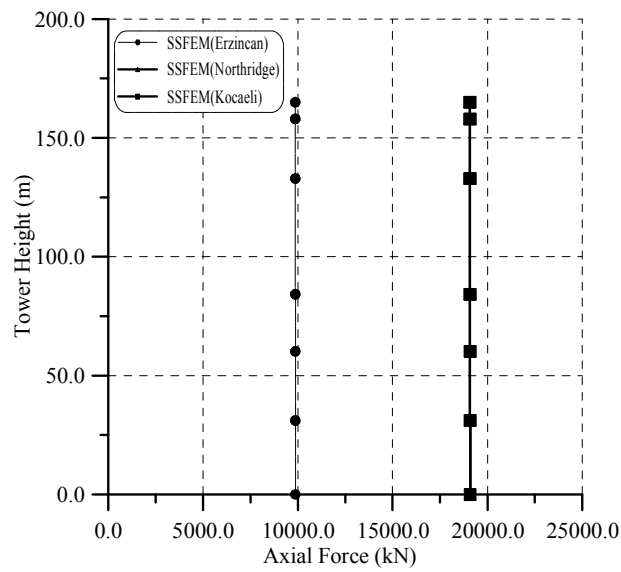
Fig. 9 Maximum sensitivity (a) axial forces; (b) shear forces and (c) bending moment for deck of Bosphorus Bridge for random elastic module (E)

absolute differences between these two methods for sensitive vertical displacement values are about 3.65%, 2.24% and 3.30%, for Kocaeli, Erzincan and Northridge NF ground motions respectively.

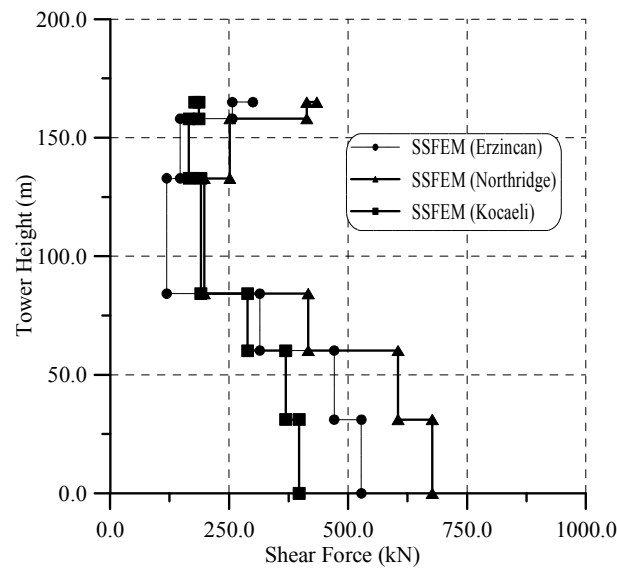
By comparing SSFEM and MCS methods gives closer results to each other. Therefore, other stochastic sensitivity internal forces (axial forces, shear forces and bending moments) are given for only SSFEM. The maximum axial forces, shear forces and bending moments at the deck of the Bosphorus Bridge obtained from sensitive analysis subjected to each ground motion are presented Figs. 9(a)-(c). The properties of the ground motions are given in Table 1. The time histories for

acceleration and velocity of records are presented in Fig. 2. The maximum axial forces and bending moment obtained from Kocaeli earthquake ground motion are the highest but the maximum shear forces obtained from Northridge earthquake.

Fig. 10 presents sensitivity axial forces, shear forces and bending moments along tower height for Bosphorus European according to SSFEM for NF Ground Motions. The maximum sensitivity axial forces and bending moment obtained from Kocaeli earthquake ground motion are the



(a)



(b)

Fig. 10 Continued

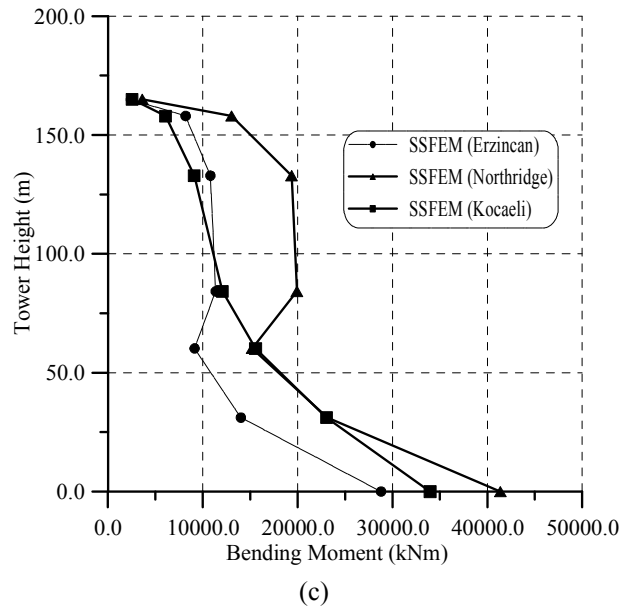


Fig. 10 (a) Maximum axial forces; (b) shear forces and (c) bending moment Bosphorus European for tower for random elastic module

highest but the maximum shear forces obtained from Northridge earthquake. It can easily be comprehended by the figures that maximum sensitivity internal forces, which are obtained from Bosphorus European tower, are the highest for Northridge 1994 earthquake ground motion and these values subjected to Kocaeli 1999 earthquake ground motion are higher than the ones subjected to Erzincan 1992 earthquake ground motion.

#### 4.2 Example 2

The Humber Bridge, which was completed in 1981, held the record for the longest span in the world. The Humber Bridge, near Kingston upon Hull, England, is a 2220 m single-span suspension bridge, the fifth-largest of its type in the world. It spans the Humber (the estuary formed by the rivers Trent and Ouse) between Barton-upon-Humber on the south bank and Hessle on the north bank, connecting the East Riding of Yorkshire and North Lincolnshire. Distances between towns in the new county would be reduced by as much as 50 miles and, with its approach roads, the Humber Bridge would form part of an integrated road system on both banks of the river connecting East Yorkshire and North Lincolnshire to the national motorway network. The bridge has a main span of 1410 m and two side spans of 280 m and 530 m on the Hessle and Barton sides, respectively (Brownjohn *et al.* 1987). The horizontal distance between the cables is 28 m and the roadway at the mid-span of the bridge is approximately 30 m above the Severn River. General view and arrangement of the Humber Suspension Bridge are shown in Figs. 11-12. In addition, material and sectional properties such as main cable, back stay cable, hangers, deck etc. of Humber Suspension Bridge were shown in Table 3.





Fig. 11 View of Humber Suspension Bridge

Table 3 Structural material and sectional properties of Humber Suspension Bridge

Members	Elastic modulus (kN/m <sup>2</sup> )	Cables sectional area (m <sup>2</sup> )	Moments of inertia (m <sup>4</sup> )	Poisson's ratio	Mass density (ton/m <sup>3</sup> )
Deck	$2.0 \times 10^8$	0.73	1.94	0.3	13.0747
Hanger	$1.40 \times 10^8$	$4.2 \times 10^{-3}$	$2.068 \times 10^{-8}$	0.3	7.8129
Main cable	$1.93 \times 10^8$	0.58	0.0134	0.3	8.5972
Backstay cable	$1.93 \times 10^8$	0.62	0.0153	0.3	9.5228
Tower	$2.0 \times 10^8$	40.75	132.046	0.3	2.400

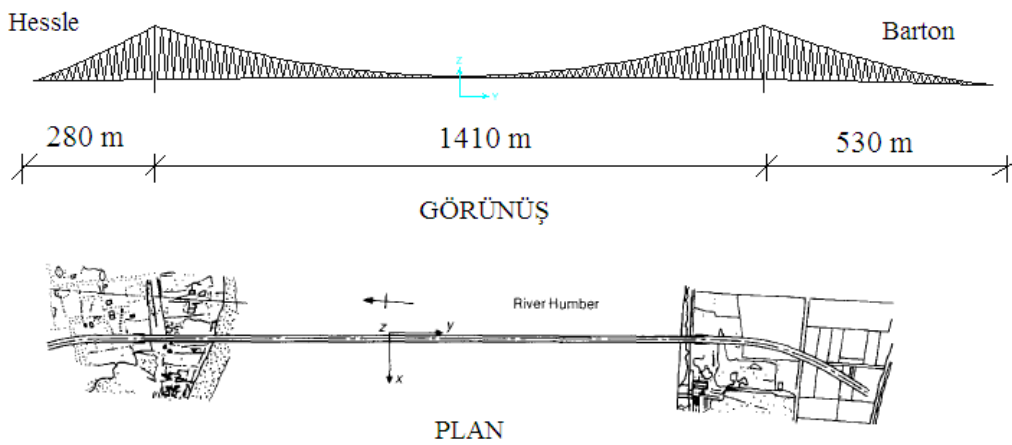


Fig. 12 Continued

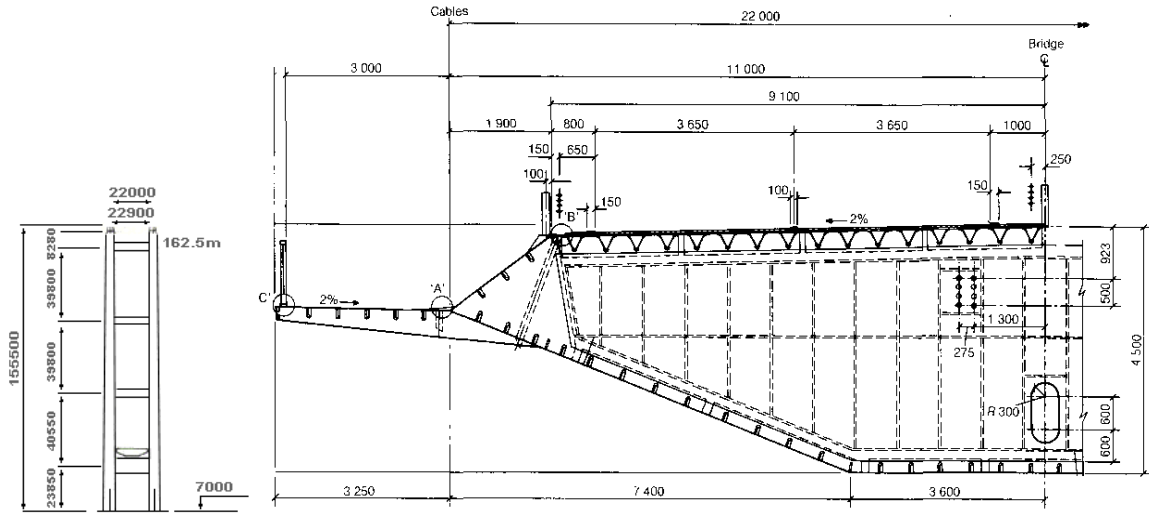


Fig. 12 General Arrangement of Humber Suspension Bridge

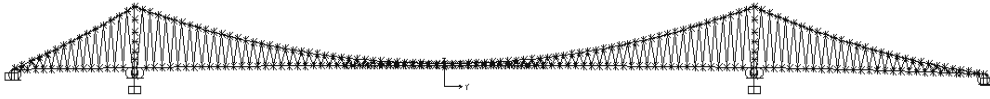


Fig. 13 Two-dimensional finite element model of Humber Suspension Bridge

To investigate the stochastic sensitivity response of the Humber Suspension Bridge under to near fault ground motion, two-dimensional mathematical model are used for calculations. The finite element model of Humber Suspension Bridge is shown in Fig. 13. As the deck, towers and cables are represented by beam elements, the hangers are represented by truss elements in the model. Because the side span decks are not connected to the cable, they are not considered in the finite element models. Finite element model of the Humber Suspension Bridge with inclined hangers has 291 nodal points, 287 beam elements and 236 truss elements and the model is represented by 855 degrees of freedom. This model has three degrees of freedom at each nodal point, namely, two translational degrees of freedom in vertical and longitudinal axes and one rotational degree of freedom in lateral axis. The suspension bridge is modeled by 523 stochastic finite elements of different length.

When the time-instant sensitivity response of the structural system of Fig. 12. The structural response functional is defined as

$$\varphi(\tau) = \frac{[q_{\beta}(\tau)]^2}{(q_{\beta}^{(A)})^2} - 1 \leq 0$$

where  $q_{\beta}(\tau)$  is vertical displacement at the apex A and  $q_{\beta}^{(A)}$  is an admissible displacement value. The elastic module is assumed to be random design variables.

Elastic module from material properties is chosen as random variable for steel frame system.

The other variables are considered as deterministic. This random variable is assumed to follow a normal distribution with the coefficient of variation 0.10. The respective expectation and correlation function and coefficient of variation for the elastic modulus  $E_\rho$  are assumed as follows

$$E[E_\rho] = 2.1 \times 10^8 \quad \lambda = 10$$

$$\mu(E_\rho, E_\sigma) = \exp\left(-\frac{|x_\rho - x_\sigma|}{\lambda l}\right) \quad \rho, \sigma = 1, 2, \dots, 523 \quad (10)$$

$$\alpha = 0.10$$

where  $x_\rho$ ,  $l$  and  $\lambda$  are ordinates of the element midpoints ( $n$  random variable,  $\rho, \sigma = 1, 2, \dots, n$ ), structural member length and decay factor, respectively. The Humber suspension bridge is modeled by 523 stochastic finite elements with different lengths. MCS method is simulated for 10000 simulations.

In the second part of this study, Humber Bridge stochastic sensitivity responses with respect to random elastic modulus according to SSFEM and MCS methods are determined compared with each other. The maximum stochastic sensitivity vertical displacement responses of the bridge deck and horizontal displacements along the Humber Hessle tower obtained from SSFEM and MCS methods for near-fault ground motion are presented in Figs. 14(a)-(b). Two analyses give very close results. The maximum stochastic sensitivity vertical displacements at deck point for Kocaeli, Erzincan and Northridge near-fault (NF) ground motions occurred as 277.65 cm, 81.62 cm and

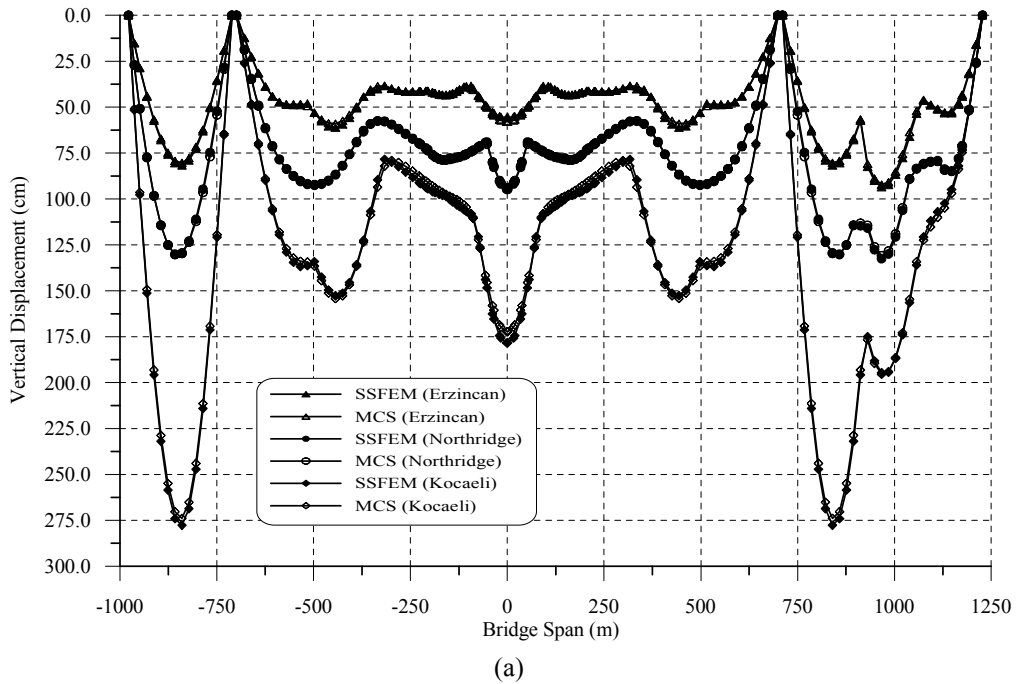


Fig. 14 Continued

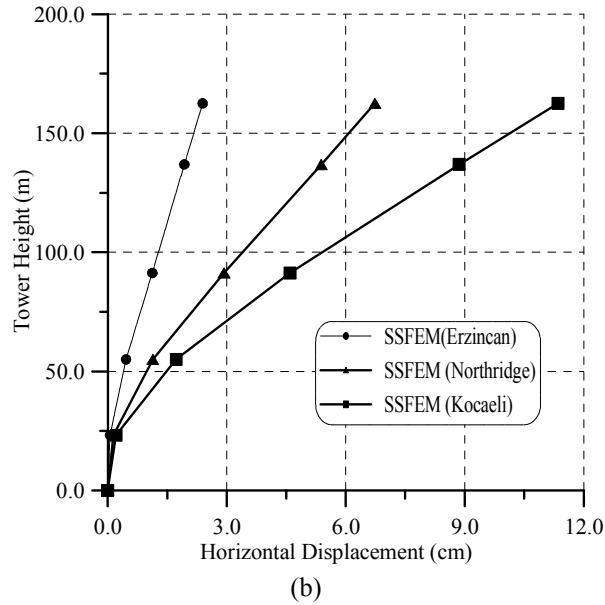


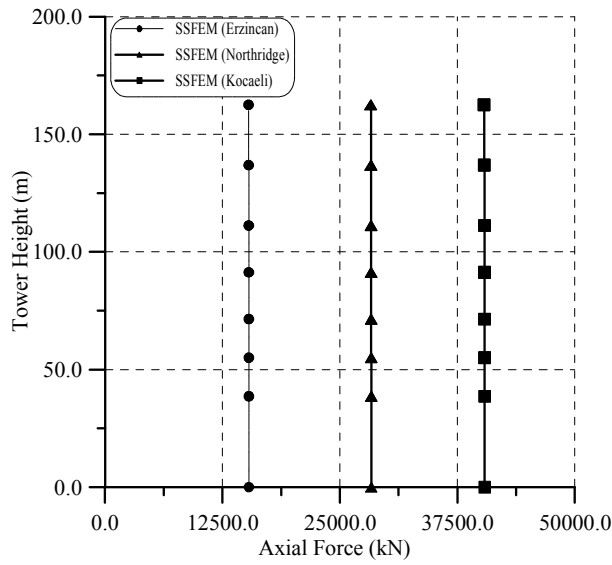
Fig. 14 Maximum sensitivity vertical displacements at the deck of Humber Bridge (a) and maximum horizontal displacements along Humber Hesse tower (b) for random elastic modulus

129.36 cm, respectively. The maximum stochastic sensitivity horizontal displacements at tower for Kocaeli, Erzincan and Northridge NF ground motions occurred as 11.35 cm, 2.39 cm and 6.74 cm, respectively. It is clearly seen that stochastic sensitivity values of vertical and horizontal displacements obtained from Kocaeli earthquake ground motion are the highest. Because Kocaeli ground motion has the peak acceleration value, maximum values of sensitivity internal forces are obtained from Kocaeli earthquake ground motion. In addition, the maximum stochastic sensitivity vertical and horizontal displacements attained from Erzincan 1992 earthquake ground motion are the lowest. By comparing SSFEM and MCS methods gives closer results to each other. The average absolute differences between these two methods for stochastic sensitivity vertical displacement values are about 2.67%, 1.56% and 2.43%, for Kocaeli, Erzincan and Northridge NF ground motions respectively.

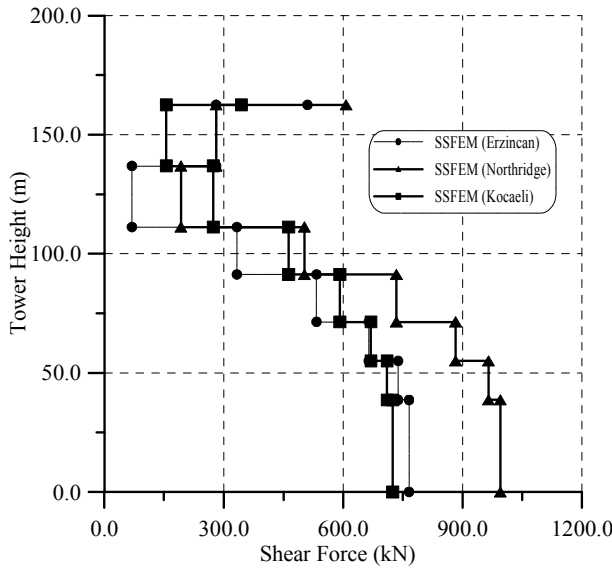
It can be seen from these figures that, the maximum values of stochastic sensitivity dynamic responses for random elastic module are very similar to the result from the MCS method. For accurate dynamic responses, it is necessary that the analysis technique incorporate the effect of structural parameter randomness. This is of special importance for accurate sensitive stochastic dynamic analysis of complex systems, which exhibit wide dispersion in structural parameters.

By comparing SSFEM and MCS methods gives closer results to each other. Therefore, other sensitivity internal forces (axial forces, shear forces and bending moments) are given for only SSFEM. The maximum axial forces, shear forces and bending moments at the deck of the Humber Bridge obtained from sensitive analysis subjected to each ground motion are presented Figs. 15(a)-(c). The maximum stochastic sensitivity axial forces, shear forces and bending moment obtained from Kocaeli earthquake ground motion are the highest at the deck of the Humber Bridge. However, the maximum stochastic sensitivity axial forces, shear forces and bending moment

obtained from Northridge earthquake ground motion are the more high for Humber Hesse tower. It can easily be comprehended by the figures below that maximum sensitive displacement and internal forces, which are obtained from Humber suspension bridge, are the highest for Kocaeli 1999 earthquake ground motion and these values subjected to Northridge 1994 earthquake ground motion are higher than the ones subjected to Erzincan 1992 earthquake ground motion. The



(a)



(b)

Fig. 16 Continued

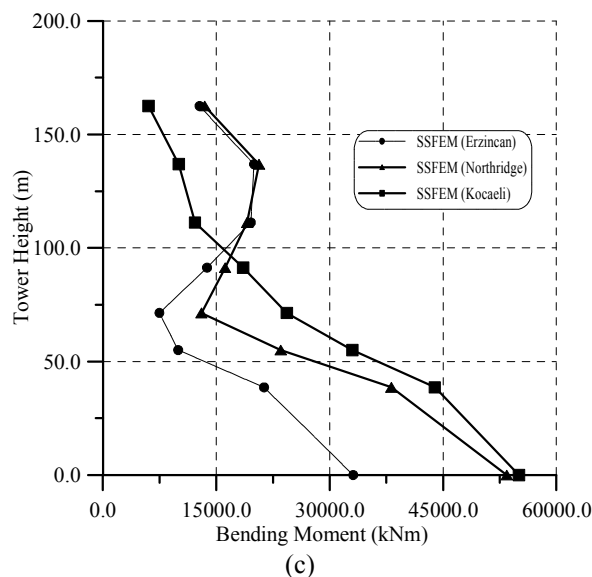


Fig. 16 Maximum axial forces (a), shear forces (b) and bending moment (c) Humber Hessle for tower for random elastic module

maximum stochastic sensitivity dynamic responses for Humber Hessle tower are given in Figs. 16(a)-(c) subjected to each ground motion. It is inferred from the below figures that while maximum sensitivity internal forces, which are obtained from Humber Hessle tower, in Northridge earthquake ground motion, minimum sensitivity internal forces are revealed in Erzincan earthquake ground motion.

In this study, at the decks of Bosphorus and Humber suspension bridges, maximum stochastic sensitivity displacements and internal forces obtained from Kocaeli earthquake ground motion are generally the highest. However, at the towers of Bosphorus and Humber suspension bridges, maximum stochastic sensitivity internal forces obtained from Northridge earthquake ground motion are generally the more high.

If it is mentioned the other results obtained from these examples; for the analysis of Bosphorus Suspension Bridge system whose numerical properties are presented (Fig. 4), it needs about 5 min for sensitivity stochastic dynamic analysis subjected to NF ground motion, however, it needs about 8 h for MCS analysis with the PC which have Intel Pentium (R) 2.40 GHz CPU and 768 MB RAM. The sensitivity analysis of Humber Suspension Bridge system whose numerical properties are given (Fig. 8), it needs about 8 min for SSFEM, however, it needs about 12 h for MCS analysis for 10000 simulations.

The stochastic sensitivity responses obtained show that selected correlation function suitable for this example for chosen coefficient of variation (COV) value ( $\alpha = 0.10$ ).

The examples clearly demonstrate the efficiency, robustness and desirability application potential of the proposed SSFEM-based algorithm. The algorithm can be used routinely for the stochastic sensitive analysis and design of the complex suspension bridges as an alternative to the currently available methods.

## 5. Conclusions

The effects of the near-fault strong ground motions on the stochastic sensitivity behaviors of suspension bridges by using Stochastic Sensitivity Finite Element method (SSFEM) and Monte Carlo simulation (MCS) were studied in this paper. Stochastic sensitivity analyses were performed on Bosphorus and Humber suspension bridges. For each bridge model, Kocaeli (1999), Northridge (1994), and Erzincan (1992) near-fault strong ground motion records that have close amplitudes were taken into account separately. The stochastic sensitivity displacements and internal forces are calculated and compared with each other.

This study confirms the importance of the ground motion selection for the accurate evaluation of the sensitive seismic performance of suspension bridges. It should be clarified that the near-fault ground motion effects appear for the duration of the earthquake. It is also seen that maximum stochastic sensitivity displacements and dynamic responses have not occurred at anytime when near-fault earthquake has peak acceleration value.

The presented numerical technique is well suited for computer-aided analysis for structural systems. The SSFEM is very effective as it provides sufficient accuracy for a small range of chosen coefficient of variation (COV). The suspended bridges modeled in this study, SSFEM gives close results to MCS method for sensitivity displacements. The numerical applications in this study are shown that the SSFEM is able to provide at an attractive computational cost, a good estimation of the sensitivity response variability.

In this study, Bosphorus and Humber suspension bridges, maximum stochastic sensitivity displacements and internal forces obtained from SSFEM are generally the high.

According to this study, the earthquake record of the near-fault ground motion, forming of the combination of numerous waves, has remarkable effect on the stochastic sensitive earthquake response of the suspension bridges. It is seen from the conclusions of this study that different near-fault strong ground motion records should be considered in the stochastic sensitivity dynamic analysis of complex suspension bridges.

Finally, the stochastic sensitivity analysis can identify the degree of robustness of the final design with respect to randomness of selected system parameters. This information can be used to determine whether system parameters uncertainty should be considered explicitly in the structural design process. The stochastic sensitivity information provides a deeper insight into the structural design and it can be used as a basis for decision-making.

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