Fifth International Conference on Analysis and Applied Mathematics

ABSTRACT BOOK

of the conference ICAAM 2020

Edited by

Prof. Dr. Charyyar Ashyralyyev

and

Prof. Dr. Evren Hincal

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The conference is organized biannually. Previous conferences were held in Gumushane, Turkey in 2012, Shymkent, Kazakhstan in 2014, Almaty, Kazakhstan in 2016, Lefkoşa (Nicosia), Mersin 10, Turkey in 2018 and in 2020. The aim of the International Conference on Analysis and Applied Mathematics (ICAAM) is to bring mathematicians working in the area of analysis and applied mathematics together to share new trends of applications of mathematics. In mathematics, the developments in the field of applied mathematics open new research areas in analysis and vice versa. That is why, we plan to found the conference series to provide a forum for researches and scientists to communicate their recent developments and to present their original results in various fields of analysis and applied mathematics.

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FOREWORD

The Organizing Committee of ICAAM and Near East University, Lefkosa (Nicosia), Mersin 10, Turkey are pleased to invite you to the Fifth International Conference on Analysis and Applied Mathematics, ICAAM 2020. The meeting will be held on September 23-30, 2020 in North Cyprus, Turkey.

The conference is organized biannually. Previous conferences were held in Gumushane, Turkey in 2012, in Shymkent, Kazakhstan in 2014, and in Almaty, Kazakhstan in 2016, Lefkoşa (Nicosia), Mersin 10, Turkey in 2018. The proceedings of ICAAM 2012, ICAAM 2014, ICAAM 2016 and ICAAM 2018 were published in AIP (American Institute of Physics) Conference Proceedings. Near East University is pleased to host the fifth conference which is focused on various topics of analysis and its applications, applied mathematics and modeling.

The conference will consist of plenary lectures, mini symposiums and contributed oral presentations.

The proceedings of ICAAM 2020 were published in AIP Conference Proceedings, too.

Selected full papers of this conference will be published in peer-reviewed journals.

The aim of the International Conference on Analysis and Applied Mathematics (ICAAM) is to bring mathematicians working in the area of analysis and applied mathematics together to share new trends of applications of mathematics. In mathematics, the developments in the field of applied mathematics open new research areas in analysis and vice versa. That is why, we plan to found the conference series to provide a forum for researches and scientists to communicate their recent developments and to present their original results in various fields of analysis and applied mathematics. The Conference Organizing Committee would like to thank our sponsors. The main organizer of the conference is Near East University, Lefkosa (Nicosia), Mersin 10, Turkey. The conference is also supported by Institute of Mathematics and Mathematical Modeling, Almaty, Kazakhstan, Analysis & PDE Center, Ghent University, Ghent, Belgium. We would like to thank Near East University, Turkey, Institute of Mathematics and Mathematical Modeling, Kazakhstan and Analysis & PDE Center, Ghent University, Ghent, Belgium for their support. We also would like to thank to all Invited Speakers, Co-Chairs, Co-ordinating Committee, International Organizing Committee, International Organizing Committee, and Technical Program Committee Members. With our best wishes and warm regards,

Chairs:

Prof. Allaberen Ashyralyev

Prof. Makhmud Sadybekov

CONTENTS

ANALYSIS

On Reich - type contraction in cone pentagonal metric spaces over Banach algebras

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Abstract: In the present paper, we studied and proved a Reich - type fixed point theorem in Cone pentagonal metric spaces over Banach algebras which extend and generalized the results in [1], [2], [3], and many well-known results in the literature. Some examples were also given to elucidate our results:

Let $(\mathcal{X}, \rho)$ be a complete cone pentagonal metric space over Banach algebra $\mathcal{A}$ with a unit element e and K be a solid cone in $\mathcal{A}$. Suppose that $F : \mathcal{X} \to \mathcal{X}$ is a mapping satisfying the following condition:

$$\rho(Fu, Fv) \preccurlyeq \vartheta_1 \rho(u, v) + \vartheta_2 \rho(Fu, u) + \vartheta_3 \rho(Fv, v), \text{ for all } u, v \in \mathcal{X},$$

where $\vartheta_j \in K$ $(j = 1, 2, 3)$ such that ϑ_1 commutes with $\vartheta_2 + \vartheta_3$, ϑ_2 commutes with ϑ_3. Then F has a unique fixed point u^* in $\mathcal{X}$. Moreover, for any point $u_0 \in \mathcal{X}$, the iterative sequence $\{F^k u_0\}$ $(k \in \mathbb{N})$ converges to u^*.

Keywords: Cone pentagonal metric space, Banach algebras, contraction mapping, fixed point

2010 Mathematics Subject Classification: 47H10, 54H25

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Chatterjea - type fixed point theorem on cone rectangular metric spaces with Banach algebras

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Abstract: In this paper, we established the prove of a new Chatterjea - type fixed point theorem in Cone rectangular metric spaces over Banach algebras as follows:

Let (K, η) be a complete cone rectangular metric space over Banach algebra B with a unit e and S be a solid cone in B. Suppose that the mapping $J : K \to K$ satisfies the contractive condition:
$$\eta(Jx, Jy) \preccurlyeq \gamma[\eta(Jx, y) + \eta(Jy, x)],$$
for all $x, y \in K$, where $\gamma \in S$ such that the spectral radius $\delta(\gamma) < 1/2$. Then J has a unique fixed point x_* in K. Moreover, for any point $x_0 \in K$, the iterative sequence $\{J^i x_0\}$ $(i \in \mathbb{N})$ converges to x_*.

Keywords: Cone rectangular metric space, contraction, fixed point, Banach algebras

2010 Mathematics Subject Classification: 47H10, 54H25, 35J08, 35J25

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Impact of working capital management on profitability of firms in Pakistan

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Abstract: Working capital management is a very important component of corporate finance because it directly affects the liquidity and profitability of the firm. This study investigates a relationship between working capital and profitability of firms. In this research we have selected a sample of 25 Pakistani firms listed on Karachi stock Exchange for a period of 6 years from 2001-2006, we have studied the effect of different variable cash, inventory, account receivable, and account payable on the profitability of Pakistani firms. The relationship between working capital management and profitability is examined using correlation and regression analyses. I have found a significant positive relationship between profitability inventory and cash. These results suggest that managers can create value for their shareholder by increasing account receivable and inventories. The negative relationship between account payable and profitability is consistent with the view that less profitable firms wait longer to pay their bills.

Keywords: Profitability, cash, inventory, accounts payable and accounts receivable.

An econometric analysis of the impact of foreign direct investment and stock market on economic growth in Nigeria (1986-2018). Vector autoregressive modeling approach (VAR).

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Abstract: This study examines the Impact of Foreign Direct Investment and Stock Market on Nigeria's economic growth. Using time series data for the period 1986 to 2018. The focal point being the role of the stock market development as a channel through which Foreign Direct Investment could promote growth. Employing an unrestricted vector autoregressive model (VAR), through impulse response and variance decomposition approach to determine shocks among the variables under study. The major contribution of this research to the literature was on proving real shocks effect of FDI and stock market on economic growth in Nigeria. Empirical estimates showed that over the period of analysis, stock prices Index (SPI) contributed about 26.4% variation (contribution) in per-capita GDP. Foreign direct investment (FDI) contributed 30.2%, inflation rate has contributed about -12.6% variation (negative contribution) to per-capita GDP, exchange rate (EXR), contributed 15.5% variation in per-capita GDP, while interest rate (INT) have contributed about 9.6% variation in per-capita GDP. Therefore, summing the positive contributed variation, the variables contributed about 81.7% variation in per-capita gross domestic product (GDP). Hence within the period of research Foreign Direct Investment and Stock Market activities had more positive influence on economic growth in the country. Therefore, the study recommended that, government should provide policies to attract more foreign investors including favorable exchange Rate, Interest Rate and Inflation Rate (Monitoring and Miscellaneous Provisions). Moreover, provision of proactive promotion of FDI into the country which will result to the implementation of the IMF monitored-liberalization of the economy with foreign investment into the manufacturing sector.

Keywords: Foreign direct investment (FDI), stock market, economic growth, VAR

Empirical modelling of money shocks, price and output fluctuations in South Africa

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Abstract: This paper seeks to examined the nature of interaction between money shocks, price and output in South Africa using time series data from 1980 to 2018. The study used the vector autoregressive model, impulse response function and variance decomposition analytical methods. The results found that price and output react positively to money shocks and that money shocks significantly influenced price and output. Money supply constitute a greater shock to output. The consequence is that money expansion has higher impact on output than on price. The findings confirm the assertation that monetary policy is a formidable instrument for price stabilization especially for South Africa that has experienced quite a stable inflation rate for sometime.

Keywords: Money shocks, price, output, impulse response function, variance decomposition

Wavelets analysis of energy consumption and economic growth on environmental degradation through ecological footprint in USA

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Abstract: his study tends to provide a new insight into the environmental literature by using the wavelets tools to examine the role of energy consumption and economic growth on environmental degradation through ecological footprint, covering the time period from 1961 to 2014. The continuous wavelet (CWT) power spectrum shows the variation (change of behaviour) of each variables over time and for different time scales; while wavelet coherence approach shows the dynamic of correlations and causality between two time-series variable over time and for different time scales. The result indicated that in the short, medium and long run, energy consumption has a positive influence over ecological footprint. Moreover, the result of causality test shows that unidirectional causality running from energy consumption to ecological footprint is found. This evidently reveals that environmental degradation is caused by economic growth and energy consumption. The recommendations provided as based on the study's outcome.
Keywords: Economic growth, energy consumption, ecological footprint, wavelets tools

Domain of generalized Riesz difference operator of fractional order in Maddox's space $\ell(p)$

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Abstract: Let $\Gamma(x)$ denotes the gamma function of a real number $x \notin \{0, -1, -2, \ldots\}$. Then the difference matrix Δ^{Bq} of fractional order q is defined as

$$(\Delta^{Bq} v)_i = \int_{l=0}^{\infty} (-1)^l \frac{\Gamma(q+1)}{l! \Gamma(q-l+1)} v_{i-l}.$$

In this paper we introduced paranormed Riesz difference sequence space $\mathbf{r}^t(\Delta^{Bq})$ of fractional order q obtained by the domain of generalized backward fractional difference operator $R^t \Delta^{Bq}$ in Maddox's space $\ell(p)$. We investigate certain topological properties and obtain the Schauder basis of the space $\mathbf{r}^t(\Delta^{Bq})$. We also obtain the $\alpha-$, $\beta-$ and $\gamma-$duals and characterize certain matrix classes related to the space $\mathbf{r}^t(\Delta^{Bq})$.

2010 Mathematics Subject Classification: 46A45,46B45

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On transmutation operators in the theory of boundary value problems

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Abstract: This report is devoted to Fredholm properties of the equation

$$(Au)(x) = v(x), \quad x \in M,$$

where M is a manifold with a non-smooth bounday, A is a pseudo-differential operator with the symbol $\tilde{A}(x, \xi)$, the right hand side $v(x)$ and the unknown function $u(x)$ belong to appropriate Sobolev–Slobodetskii spaces.

We suggest some constructions to develop the theory of boundary value problems on manifolds with a non-smooth boundaries. We discuss basic principles for such a theory and describe main results that we have obtained to this time. Further, we show how these results are related to the theory of boundary value problems on non-smooth manifolds.

Special operators, so-called transmutation operators play an important role in these considerations. Some of them were described in our papers [1,2]. We can extend the class of such operators.

Keywords: elliptic pseudo-differential equation, wave factorization, general solution, boundary value problem

2010 Mathematics Subject Classification: 35S15, 47G30

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Completness of roots elements of linear operators in Banach spaces and applications

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Abstract: Here, the spectral properties of linear operators in Banach spaces are studied. We find sufficient conditions on structure of Banach spaces and resolvent properties that guarantee completeness of roots elements of Schatten class operators. This approach generalizes the well known result for operators in Hilbert spaces [1, Theorem 9.29]. The articles [2], [3] and [4] are devoted to this question in Banach spaces. In this paper, we disclose different sufficient condition for completeness of roots elements of linear operators. We consider the class of Banach spaces which satisfy some given conditions, but our class are wider than the class of operators considered in [2], [6] and [15]. Moreover, our method of proofs are different from proofs provided in the cited references. We find a sufficient condition on structure of Banach spaces which allow to define the trace of operators and its properties. Also, we get Carleman estimates and spectral properties of linear operators possess the quasi nuclear inverses. In application we consider nonlocal boundary value problem (BVP) for the second order differential-operator equation with top variable coefficients

$$Lu = a\left(x\right)u^{(2m)}\left(x\right) + \sum_{k=0}^{2m-1} B_k\left(x\right)u^{(k)}\left(x\right) + A\left(x\right)u\left(x\right) = f\left(x\right),\ x \in \left(0,1\right),$$

where a is a complex-valued function, $A\left(x\right)$, $B_k\left(x\right)$ are linear operators in a Banach space E and f is a E-valued function. In application, the boundary value problems for the abstract differential equations with variable coefficients are studied. The principal parts of the appropriate differential operators are not self-adjoint. The discreetness of spectrum and completeness of root elements of these operators are obtained.

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Fourier multipliers and embedding theorems in Sobolev-Lions type spaces and application

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Abstract: In this talk, Mikhlin and Marcinkiewicz–Lizorkin type operator-valued multiplier theorems in weighted abstract Lebesgue spaces are studied. Using these results one derives embedding theorems in E_0-valued weighted Sobolev-Lions type spaces $W_{p,\gamma}^l(\Omega; E_0, E)$, where E_0, E are two Banach spaces, E_0 is continuously and densely embedded into E. We prove that, there exists a smoothest interpolation space E_α, between E_0 and E, such that the differential operator D^α acts as a bounded linear operator from $W_{p,\gamma}^l(\Omega; E_0, E)$ to $L_{p,\gamma}(\Omega; E_\alpha)$ and the following Ehrling-Nirenberg-Gagilardo type sharp estimate holds

$$\|D^\alpha u\|_{L_{p,\gamma}\left(\Omega; E\left(A^{1-|\alpha:l|-\mu}\right)\right)} \le C_\mu \left[h^\mu \|u\|_{W_{p,\gamma}^l(\Omega; E(A), E)} + h^{-(1-\mu)} \|u\|_{L_{p,\gamma}(\Omega; E)} \right]$$

for $u \in W_{p,\gamma}^l(\Omega; E(A), E)$. Finally, we consider the abstract differential equation

$$(1) \qquad Lu = \sum_{|\alpha|=2l} a_\alpha D^\alpha u + Au + \sum_{|\alpha|<2l} A_\alpha(x) D^\alpha u + \lambda u = f,$$

where a_α are complex numbers, A, $A_\alpha(x)$ are linear operators in a Banach space E and λ is a complex parameter.

We show that there exists a unique solution $u \in W_{p,\gamma}^{2l}(\mathbb{R}^n; E(A), E)$ to (1) for all $f \in L_{p,\gamma}(\mathbb{R}^n; E)$ and there exists a positive constant C depend only on p and γ such that the following coercive uniform estimate holds

$$\sum_{|\alpha|\le 2l} |\lambda|^{1-\frac{|\alpha|}{2l}} \|D^\alpha u\|_{L_{p,\gamma}(\mathbb{R}^n; E)} + \|Au\|_{L_{p,\gamma}(\mathbb{R}^n; E)} \le C \|f\|_{L_{p,\gamma}(\mathbb{R}^n; E)}.$$

By using the separability properties of (2) we show that the corresponding Cauchy problem for the parabolic equation

$$(2) \qquad \partial_t u + \sum_{|\alpha|=2l} a_\alpha D^\alpha u + Au = f(t,x),\ t \in (0,\infty),\ x \in \mathbb{R}^n,$$

$$u(0,x) = 0,\ x \in \mathbb{R}^n$$

is well-posed in weighted spaces $L_{\mathbf{p},\gamma}(\mathbb{R}^n; E)$ with mixed norm, where $\mathbf{p} = (p, p_1)$.

The Riemann -Hilbert problem for first order elliptic systems on the plane in the Hardy space on the plane in the Hardy space

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Abstract: Let D be a finite domain bounded by a smooth Lyapunov contour $\Gamma \in C^{1,\nu}, 0 < \nu < 1$, which is oriented positively with respect to D. Let all eigenvalues of a matrix $J \in \mathbb{C}^{l \times l}$ lie on the upper half-plane. and $G(t)$ be Holder continuous $l \times l-$ matrix- value function on Γ such that $\det G(t) \neq 0$ for all $t \in \Gamma$.

We consider the Riemann – Hilbert problem

$$\frac{\partial \phi}{\partial y} - J\frac{\partial \phi}{\partial x} = 0, \quad z = x + iy \in D,$$

$$\text{Re } G(t)\phi^+(t) = f(t), \quad t \in \Gamma,$$

in the the Hardy- Smirnov space $H^p(D)$, $1 < p < \infty$, which is defined [1] analogously to the case of usual analytic function.

It is shown that this problem is Fredholmian and its index æ given by the formula

$$æ = -\frac{1}{\pi} \arg \det G(t)\big|_\Gamma + l.$$

With the help of this result the integral representation of all solutions $\phi \in H^p(D)$ of (1) is received.

Keywords: First order elliptic system, Riemann- Hilbert problem, Hardy space, index formula, integral representation

2010 Mathematics Subject Classification: 35J56, 30D60, 30E25, 30H10

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APPLIED MATHEMATICS

Optimal control of a chemovirotherapy model

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Abstract: Chemovirotherapy has been developed as a new way of combination therapy used to treat various types of cancer. A mathematical model of chemovirotherapy is adopted [1], with the objective of applying optimal control theory to find optimal doses of the combined treatment required to neutralize the cancer with minimal toxicity. Two control functions are incorporated into the model, the first one represents optimal dose of chemotherapeutic agent while the second one signifies the optimal dose of oncolytic virus to be injected into the patient's body. Pontryagin's maximum principle is followed to get the characterization of the optimal control pair. A fourth order Runge-Kutta algorithm is used to obtain the numerical solutions of the optimality system which is a two-point boundary value problem. Existence and uniqueness theorem of the optimality system is stated. Numerical simulations show that when the optimal control pair is applied the cancer cells are eventually decreased. Thus, the optimal doses obtained should be espoused in order to successfully eliminate the cancer.

Keywords: Chemovirotherapy, optimal control, Pontryagin's maximum principle, Hamiltonian, optimality system

2010 Mathematics Subject Classification: 35J05, 35J08, 35J25

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Theoretical study of the Duffing and Van Der Pol oscillators as a stochastic differential equation

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Abstract: In this work, we will focus on the theoretical study of Duffing and Van Der Pol oscillators, such as the existence, uniqueness and non-explosion of solutions, we start by modeling these two oscillators as, an stochastic differential equation, then based on a non-explosion criterion of Khasminski, we will build Lyapunov functions which ensure the non-explosion of solution, where we will obtain sufficient conditions so that the solution does not explode.

Keywords: Duffing, Van Der Pol, non-explosion, SDE, Lyaponov function

2010 Mathematics Subject Classification: 60H10, 60J70, 34C15

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A numerical algorithm for the involutory parabolic problem with Neumann condition

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Abstract: Parabolic and elliptic type partial differential equations with involution in x have been investigted in papers [1]-[2]. In [3], the theory of the basis property of eigenfunctions of second order differential operators with involution was investigated, on this basis the Fourier method was justified for solving direct and inverse problems for one dimentional parabolic equations with involution in x. The existence and uniqueness of the solution of a mixed problem for a parabolic equation with an involution in x in the form of a Fourier series were established. The questions of solvability of inverse problems for the heat equation and their fractional analogues were investigated. Solvability of inverse problems for a parabolic equation with an involution in x was proved. In the present paper, the first and second order of accuracy difference schemes for the numerical solution of the initial boundary value problem for one dimensional parabolic type involutory partial differential equation with Neumann condition are presented. Numerical results are provided.

Keywords: Involutory parabolic equation, difference schemes, numerical experience

2010 Mathematics Subject Classification: 35J25, 47E05, 34B27

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Soret effect on unsteady free convection heat and mass transfer flow in a vertical channel with ramped wall temperature and concentration

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Abstract: This study presents analytical solutions to unsteady natural convection heat and mass transfer flow in a vertical channel in the presence of Soret effect. The bounding walls are considered to have ramped boundary conditions on the temperature and concentration. The system of the dimensionless governing coupled boundary layer PDEs have been solved analytically using the powerful Laplace transform technique (LTT) and numerically by constructing a program in MATLAB. Comparisons of numerical and analytical results are made while the numerical results for the temperature, concentration, velocity as well as the heat transfer rate, mass transfer rate and wall skin friction are presented graphically. The study shows that incorporating ramped BCs makes the fluid temperature, specie concentration, flow velocity, coefficient of rate of heat transfer, coefficient of rate of mass transfer and coefficient of Skin-friction to be lower compared to the case of constant BCs. Moreover, Soret effect affects the concentration and velocity.

Keywords: Soret effect, free convection, mass transfer, ramped temperature, ramped concentration, Laplace transform technique

2010 Mathematics Subject Classification: 76R10, 80A20, 44A10

On the boundedness of solution of the Schrödinger differential equation with time involution

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Abstract: In the present work, the initial value problem for the Schrödinger type involutory partial differential equation is studied. Applying Green's function of space operator, we get formula for solution of this problem. In applications,the theorem on stability of this problem is established. Furthermore, this formula is applied to the existence and uniqueness of bounded solution of nonlinear problem with involution.

Keywords: Schrödinger type differential equation, boundedness, involution

2010 Mathematics Subject Classification: 35J25, 47E05, 34B27

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On uniform difference schemes and asymptotic formulas for the solution of Shrödinger's type nonlocal boundary value perturbation problems

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Abstract: The abstract nonlocal boundary value problem

$$\begin{cases} i\varepsilon u^{'}(t) + Au(t) = f(t), 0 < t < T, \\ u(0) = \int\limits_{0}^{T} \alpha(s)u(s)ds + \varphi \end{cases}$$

for Shrödinger equations in a Hilbert space H with the self adjoint positive definite operator A and with an arbitrary $\varepsilon \in (0, \infty)$ parameter multiplying the derivative term is considered. An asymptotic formula for the solution of this problem with a small ε parameter is established. The high order of accuracy single-step uniform difference schemes for the solution of this problem are presented. The convergence estimates for the solution of these difference schemes are established.

Keywords: Asymptotic formula, uniform difference schemes, Schrödinger problem

2010 Mathematics Subject Classification: 35J25, 47E05, 34B27.

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Exact solution of Schrödinger equation in 2D deSitter and anti-deSitter Spaces for Kratzer potential plus a dipole

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Abstract: In this work we gave the exact solution of Schrödinger equation for Kratzer Potential plus a Dipole potential $V(r,\theta) = \frac{1}{4\pi\epsilon_0}\left(\frac{Q}{r} + \frac{D_r}{r^2} + \frac{D_\theta \cos\theta}{r^2}\right)$ [1] in 2D deSitter and Anti-deSitter spaces ,this deformed space defined by the following commutation relations [2] [3] $[X_i, X_j] = 0$, $[P_i, P_j] = i\hbar\tau\lambda\epsilon_{ijk}L_k$, $[X_i, P_j] = i\hbar(\delta_{ij} - \tau\lambda X_i X_j)$ with $\tau = -1, +1$ the noncommutative operators X_i and P_i satisfying the modified algebra

$$X_i = \frac{x_i}{\sqrt{1 + \tau\lambda r^2}} \, and \, P_i = -i\hbar\sqrt{1 + \tau\lambda r^2}\partial_{x_i}$$

when the deformed Schrödinger equation in polar coordinate is

$$\left[\begin{array}{c} -\frac{\hbar^2}{2\mu}\left[(1 + \tau\lambda r^2)\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\right) + \tau\lambda r\frac{\partial}{\partial r}\right] + \\ \frac{q}{4\pi\epsilon_0}\left(\frac{Q\sqrt{1+\tau\lambda r^2}}{r} + \frac{(1+\tau\lambda r^2)(D_r + D_\theta\cos\theta)}{r^2}\right) \end{array}\right]\psi = E\psi$$

After the separation we get tow equations the radial equation and the angular equation , the angular equation is Mathieu equation like and its solution is Mathieu function ,we used   Nikiforov–Uvarov method [4] to solve the radial equation

The energy eigenvalue is given in their exact forms and the corresponding radial wave functions are given in terms of Romanovski polynomials and the angular wave functions are expressed in terms of Mathieu function [5].

$$E_n = -\frac{\hbar^2}{8\mu}\left[\left(\frac{\alpha Q}{n + \frac{1}{2} + \nu}\right)^2 + 4\lambda\left(\left(n + \frac{1}{2} + \nu\right)^2 - \left(\nu^2 + \frac{1}{4}\right)\right)\right]$$

$$\Psi_n(r,\theta) = C_n^\lambda\left(\sqrt{\lambda}r\right)^{\left(\delta_1^\lambda - \frac{1}{2}\right)} e^{\frac{\eta}{2\delta_1^\lambda}\tan^{-1}\left(\frac{\sqrt{1-\lambda r^2}}{\sqrt{\lambda}r}\right)} R_n^{\left(-\delta_1^\lambda, \frac{\eta}{\delta_1^\lambda}\right)}\left(\frac{\sqrt{1-\lambda r^2}}{\sqrt{\lambda}r}\right)\Theta(\theta)$$

We have also studied the effect of the spatial deformation parameter on the bound states.

Keywords: 2D Schröding er equation Kratzer potential, Dipole potential, deSitter and Anti-deSitter spaces, Nikiforov–Uvarov method, Mathieu function

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The applications of non-polynomial spline to the numerical solution for the fractional differential equations

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Abstract: This paper presents a new computation and task on non-polynomial spline of fractional order to solve the fractional deferential equations by Caputo fractional derivative. The usual Taylor series is used to discretize the time derivative of the function. Several examine problems are present to confirm the accuracy of the spline method and to show the completion of Non-polynomial spline. In addition, we show the numerical computations provident and can be used to solve difficult problems, also the results are found to be in good error estimations with known exact solutions.

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Comparative analysis of the weighted finite element method and FEM with mesh refinement

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Abstract: The generalized solution of the boundary value problem for the Lamé system in a two-dimensional domain with a boundary containing a reentrant corner γ belongs to the space $W_2^{1+\alpha-\varepsilon}(\Omega)$, where $0.25 \leq \alpha \leq 0.63$ for $3\pi/2 \leq \gamma \leq 2\pi$, and ε is any positive number. Therefore, the approximate solution produced by classical finite element or finite difference schemes converges to a generalized solution no faster than at $O(h^\alpha)$ rate.

We defined for boundary value problems with strong and weak singularities, the solution as an R_ν-generalized one in a weighted Sobolev space or set. Relying on this approach, numerical methods were created with a rate of convergence independent of the size (value) of the singularity [1-3].

In this report we consider the Lamé system posed in a domain with the reentrant corner of $3\pi/2$. We construct a version of the weighted finite element method (FEM) on the base of a definition of the R_ν-generalized solution. We performed a comparative numerical analysis of the weighted finite element method and FEM with mesh refinement for finding an approximate solutions of model problems of elasticity theory in the L-shaped domain. A weighted FEM allows one to find a solution with theoretical accuracy on meshes of high dimension. The finite element method with mesh refinement fails at mesh high dimensions and the same calculation conditions. We carry out a comparative analysis of the absolute errors of these methods for a model boundary value problem for the Lamé system in an L-shaped domain.

Keywords: Boundary value problems with a singularity, weighted finite element method

2010 Mathematics Subject Classification: 65N30, 35Q30

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Approximation of inverse problems for fractional equations

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Abstract: This report is devoted to the approximation of inverse problem in a Banach space E

$$(\mathbf{D}_t^\alpha u)(t) = Au(t) + F(t)f, \quad t \in [0, T], u(0) = u^0, u(T) = u^T, \, 0 < \alpha < 1,$$

with operator A, which generates analytic and compact α-times resolvent family $\{S_\alpha(t, A)\}_{t \geq 0}$, function $F(\cdot) \in C^1([0, T])$, $u^0, u^T \in D(A)$ are given and $f \in E$ is unknown element.

Let A_n be a generator of analytic and compact C_0-semigroup $\exp(\cdot A_n)$. Consider in a Banach space E_n the problem

$$(\mathbf{D}_t^\alpha u_n)(t) = A_n u_n(t) + F_n(t)f_n, \quad t \in [0, T]$$

with $F_n(\cdot) \in C^1([0, T])$ and

$$u_n(0) = u_n^0 \in D(A_n), \, u_n(T) = u_n^T \in D(A_n).$$

We develop ideas of [1] and use technique of [2] to obtain the semidiscrete approximation theorem.

Keywords: Discrete equations, inverse problems, characteristic function, overdetermination, positiveness

2010 Mathematics Subject Classification: 35N10, 41A65 **References:**

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New numerical approach for solving the Oseen problem in a convection form in non-convex domain

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Abstract: An essential class of problems describes physical processes occurring in non-convex domains containing a corner greater than π on the boundary. The solution in the neighborhood of a reentrant corner is singular and its finding using classical approaches entails a loss of accuracy. In the report, we consider stationary, linearized by Picard's iterations, Navier-Stokes equations governing the flow of a incompressible viscous fluid in the convection form in non-convex polygonal domain containing one reentrant corner on its boundary. An R_ν-generalized solution of the problem in special sets of weighted spaces is defined. A weighted finite element method to find an approximate R_ν-generalized solution is constructed. The degree of the weight function, as well as the parameter ν in the definition of an R_ν-generalized solution, and a radius of a neighborhood of the singularity point are free parameters of the method. Earlier, using the proposed method, the Stokes problem and Oseen system in a rotation form were numerically studied in [1-3]. A specially selected combination of them leads to increase the convergence rate of an approximate solution to the exact one in relation to the classical approaches. The convergence rate is equal to one by the grid step in the norm of the weighted Sobolev space and does not depend on the value of the reentrant corner.

Keywords: Oseen problem with singularity, weighted finite element method

2010 Mathematics Subject Classification: 65N30, 35Q30

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Multivariate stochastic mechanisms and information measures in population growth processes

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Abstract: Stochastic differential equations have been intensively used to analyze data from physics, finance, engineering, medicine, biology, and forestry. This study proposes a general multivariate stochastic dynamical model of a population growth development which includes random forces governing the dynamic of multivariate distribution of tree size variables. The dynamic of the multivariate probability density function of individuals size variables in a stand is described by a mixed effect parameters Gompertz-type multivariate stochastic differential equation (SDE). The advantages of multivariate SDE model are that it do not need to choose many different equations to be tried, but it relates the individuals size variables dynamic against the time dimension, and consider the underlying covariance structure driving changes in the size variables. SDE model allows us a better understanding of biological processes driving the dynamic of natural phenomena. The new derived multivariate probability density function and its marginal univariate, bivariate, trivariate, conditional univariate, bivariate, trivariate, and much more distributions can be applied for the modeling of population attributes such as the mean value, quantiles and much more. This study introduces general multivariate mutual information measures based on the differential entropy to capture multivariate interactions between size variables. The purpose of the present study is therefore to experimentally confirm the effectiveness of using multivariate mutual information measures to reconstruct multivariate interactions in size variables.

Keywords: Multivariate stochastic differential equation, probability density function, maximum likelihood procedure, information measures

2010 Mathematics Subject Classification: 34F05, 60H10, 60H30

Numerical solution of the reverse parabolic problem with integral and second kind boundary condition

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Abstract: The papers [1]-[3] are devoted to study well-posedness of reverse parabolic problems with nonlocal condition and their approximations.

Let Ω be unit open cube in R^n, $S = \partial\Omega$, $\overline{\Omega} = S \cup \Omega$, and $f : (0,1) \times \Omega \to R, a_r : \Omega \to R, \mu : [0,1] \to R, \psi : \overline{\Omega} \to R$, be given functions, σ is known number ($\sigma > 0$), $\forall x \in \Omega$, $\forall r = 1, ..., n, a_r(x) \geq a_0 > 0$.

In this study, we consider approximation of the following reverse parabolic problem with integral and second kind boundary condition

(1)
$$
\begin{cases}
u_t(x,t) + \sum_{r=1}^{n} (a_r(x)u_{x_r}(x,t))_{x_r} - \sigma u(x,t) = f(x,t), \\
x = (x_1, .., x_n) \in \Omega, 0 < t < 1, \\
u(x,1) = \int_0^1 \mu(s)u(x,s)ds + \psi(x), \ x \in \overline{\Omega}; \\
\frac{\partial u}{\partial \overrightarrow{n}}(x,t) = 0, \ x \in S, 0 \leq t \leq 1.
\end{cases}
$$

We study the first order difference scheme for approximate solution of reverse problem. Stability and coercive stability estimates for solution of difference scheme are obtained. Numerical results together with an explanation on the realization in one and two dimensional test examples are presented.

Keywords: Difference schemes, reverse parabolic problem, stability, integral condition

2010 Mathematics Subject Classification: 35K60, 65M06

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The liquid-phase process of smelting reduction in an electric furnace with bottom electrodes in metallurgical DC furnaces

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Abstract: The paper devoted to the scientific bases of energy-efficient, environmentally friendly competitive smelting reduction processes, implemented in the electric furnace with two bottom electrodes. The furnace with bottom electrodes in laboratory experiments shows big industrial potential in comparison with well-known electrical furnaces such as DC arc furnaces and AC arc furnaces. The biggest advantages of such furnace are direct power supplied to the remelting materials, working without graphite electrodes, low energy losses, environmental friendly according to low dust production. The preliminary numerical simulation of this furnace and the first steps for numerical optimization and estimation of the industrial potential of such furnace are presented.

Keywords: Numerical simulation, electrovortex flow, DC metallurgical furnace

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The mathematical model of a short arc at the blow-off repulsion of electrical contacts during the transition from metallic phase to gaseous phase

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Abstract: The mathematical model describing the dynamics of temperature field in electrical contacts at the initial stage of a blow-off repulsion is presented. It is based on the Stefan problem for the disk of a short arc and two spherical domains for the liquid and solid zones. All coefficients in the equations such as the thermal and electrical conductivities, density, thermal capacity are dependent on the temperature. The analytical solution of this problem is obtained using the similarity principle. The results of calculation are compared with the data obtained in published papers and with the experimental data.

Mathematical modeling of the electrical arc is very important to understand its dynamics and to estimate arc parameters because experimental methods give as a rule only the resulting information about arcing and arc erosion because of a fleeting process. General models describing phenomena in the arc plasma are based on the systems of partial differential equations of the magneto-hydrodynamics (MHD) [1] - [4].

Keywords: Similarity principle, temperature dependence, metallic phase, gaseous phase, Stefan problem.

2010 Mathematics Subject Classification: 80A22, 35K05, 45D05

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Using the conjugate equations method for solving of inverse problems of mathematical physics and mathematical epidemiology

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Abstract: G.I.Marchuk in his work [1] dedicated to conjugate equations and their applications notes that in the XXI century the theory of conjugate equations will play an exceptional role in solving complex systems, thanks to which it will be possible to solve global problems that concern society. To study complex systems and mathematical models, a number of general approaches and principles have been developed. One of them is the principle of duality. In this principle, the concept of the conjugate operator is basic.

In general view, we write the mathematical model of the inverse problem in the operator equation form

$$(1) \qquad\qquad Au = f$$

with linear operator $A : X \to Y$ and domain $D(A)$, dense in X, i.e. $D(A) = X$. Suppose that one of the goals consideration inverse problems is to minimization of the functional of the coefficient inverse problem

$$(2) \qquad\qquad J(q, u) = \langle u, q \rangle$$

We consider the conjugate operator A^* and the conjugate equation

$$(3) \qquad\qquad A^* u^* = q$$

with element q on the right hand side defining by $J(q, u)$. Then the dual expression for the desired functional has the form

$$(4) \qquad\qquad J = \langle f, u^* \rangle$$

Thus, to calculate functional (2), it is necessary to solve the conjugate equation (3) and use formula (4). In this work, the theory of conjugate equations is used to solve the inverse problem of the continuation of potential fields in the direction of disturbing masses. This problem leads to the solution of the first kind Fredholm integral equation [2].

The next task is the inverse MTS problem [3-7]. To solve the inverse MTS problem with an additional condition for the solution, the optimal control method with the duality principle is used. The principle of duality simplifies the procedure for solving the problem. For this, a conjugate equation is introduced with the elements on the right-hand side determining the objective function of the optimal control method. Then, a gradient iterative method is constructed and a numerical solution is found.

The conjugate equations are also used to solve mathematical epidemiology problems. The paper [8] presents a methodology for the joint use of mathematical models and real data, which is an effective tool for studying complex epidemiological processes and solving practical problems based on it. A significant role here is played conjugate tasks. It is convenient to construct direct and inverse connections between models and daily real data on the distribution of the COVID - 19 coronavirus and the system organization of computing technologies using variational principles.

Keywords: Inverse problem, conjugate equation, operator equation, functional, Fredholm integral equation, mathematical geophysics, mathematical epidemiology

2010 Mathematics Subject Classification: 34M50, 49N45, 49M30,65L09

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Study of the initial boundary value problem for the two-dimensional differential equation with fractional time derivative in the sense of Caputo

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Abstract: An initial boundary value problem for a two-dimensional differential equation with a fractional time derivative in the sense of Caputo is studied. This equation is of great applied importance in modeling flow processes and anomalous dispersion.

The uniqueness and continuous dependence of the solution on the input data in differential form is proved. A computationally effective implicit scheme with weights is proposed. A priori estimates are obtained for the solution of the difference problem under the assumption that a solution exists in the class of sufficiently smooth functions. These estimates imply the uniqueness of the solution and the stability of the scheme with respect to the initial data and the right-hand side of the equation. The convergence of the approximate solution to the solution of the differential problem with the second order both in time and space variables is proved. The results of computational experiments confirming the reliability of theoretical analysis are presented.

Keywords: Fractional partial differential equation, Caputo derivative, uniqueness of the solution, a priori estimate, numerical method, stability, convergence.

2010 Mathematics Subject Classification: 35R11, 65N12, 76M20

Determination of fractional order and source term in a fractional subdiffusion equation

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Abstract: In this paper we consider an inverse problem for simultaneously determining the order of time fractional derivative and a source function in subdiffusion equations.

Keywords: Subdiffusion equation, Riemann-Liouville derivatives, inverse and initial-boundary value problem, determination of the fractional derivative's order, Fourier method.

$O(h^8 \,|\ln h|)$ order of accurate difference method for solving the Dirichlet problem for Laplace's equation on a rectangle with boundary values in $C^{k,1}$

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Abstract: A three stage (9-point, 5-point and 5-point) difference method for solving the Dirichlet problem for Laplace's equation on a rectangle is proposed and justified. It is proved that the proposed difference solution converges uniformly to the exact solution of order $O(h^8 \,|\ln h|)$, h is the mesh size, when the boundary functions are from $C^{9,1}$. Numerical experiment is illustrated to support the analysis made.

Keywords: finite difference method, error estimations, highly accurate methods

2010 Mathematics Subject Classification: 65M06, 65M12, 65M22

A fourth order accurate difference method for solving the second order elliptic equation with integral boundary condition

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Abstract: A uniform estmimation of order $O(h^4)$, h is the mesh step, for the convergence of the finite difference solution for the general second order elliptic equation with nonlocal integral boundary condition, is obtained.

Keywords: finite difference method, error estimations, highly accurate methods.

2010 Mathematics Subject Classification: 65M06, 65M12, 65M22

An algorithm for control correction in nonlinear point-to-point control problem

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Abstract: The paper proposes an algorithm for constructing a correction of an admissible control in the next nonlinear point-to-point control problem

$$\dot{x} = F_0(t, x, u) + \varepsilon F_1(t, x, u), \quad x_0(0) = x^0, \quad x_0(T) = x^T,$$
(1)
$$0 < \varepsilon \leq \varepsilon_0, \quad x \in X \subset R^n, \quad u \in U \subseteq R^n,$$

where x is a state vector, u is a control vector, ε_0 is a sufficiently small positive number, X and U are corresponding closed or open sets of permissible values of state and control vectors.

The correction consists of two terms. The first one is constructed using an auxiliary linear-quadratic control problem and has a feedback form. The second one is intended to clarify the solution and fulfill the constraints on the coordinates of the system and control. It is open-loop control and is based on the solution of the nonlinear programming problem. Note that this problem is solved in the neighborhood of the control with the first correcting term, which is already found. That reduces the solution search time. The experiments showed the effectiveness of the proposed approach.

Keywords: constrained control problem, nonlinear point-to-point control problem, perturbation method, Kalman regulator.

2010 Mathematics Subject Classification: 34E10, 34J15, 49M30, 49N35

Parallel CUDA implementation of a numerical algorithm for solving the Navier-Stokes equations using the pressure uniqueness condition

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Abstract: Two numerical methods for solving the Navier-Stokes equations in doubly connected domains are studied. The first method is based on constructing a difference problem in variables of the stream function and the vortex of velocity using the uniqueness condition for pressure. The numerical solution of the elliptic equation for stream functions is found as the sum of the solutions of two simple problems of an elliptic type. One problem is with homogeneous boundary conditions, and the other is with a homogeneous equation. An alternative approach to solving the problem is the fictitious domain method with the continuation of the least coefficient. This method does not require satisfying the pressure uniqueness condition, and is simple to implement.

A computational finite difference algorithm for solving an auxiliary problem of the fictitious domain method has been developed. The results of numerical modeling of the two-dimensional Navier-Stokes equations by the fictitious domain method with continuation by the lowest coefficient are presented. For this problem, a parallel algorithm was developed using the CUDA architecture, which was tested on various grid dimensions.

Keywords: Navier-Stokes equations, current function, vortex of velocity, multiconnected area, pressure uniqueness condition, fictitious domain method, border conditions, CUDA; parallel algorithm, high performance computing.

2010 Mathematics Subject Classification: 65M85, 35Q30, 65Y05

The fictitious domain method for the Navier-Stokes equations in natural variables

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Abstract: In this paper, we consider a variant of the fictitious domain method associated with the modification of nonlinear terms in a fictitious subdomain. The model problem shows the effectiveness of using this modification. The proposed version of the method is used to solve the problem of an arbitrary region and to set a boundary condition for pressure. A numerical solution is implemented and the results of numerical results are given.

Keywords: Fictitious domain method, Navier-Stokes equations, numerical solution.

2010 Mathematics Subject Classification: 65M85, 35Q30, 74S20

A general boundary value problem for heat and mass transfer equations with high order normal derivatives in boundary conditions

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Abstract: A Multidimensional boundary value problem of heat and mass transfer, when the boundary conditions contain higher-order derivatives was considered in [1].

In this paper, we consider the following boundary value problem:

$$(1) \qquad \frac{\partial U_k(x,t)}{\partial t} = \lambda_k \Delta U_k(x,t), \ k = 1, 2$$

in the domain $Q_T \equiv \left\{ (x', x_n, t) : x' \in R^{n-1}, x_n \in R_+, t \in]0, T[\right\}$, with the initial and boundary conditions:

$$(2) \qquad U_k(x,t) = 0,$$

$$(3) \qquad \sum_{k=1}^{2} \left[\sum_{k_n=0}^{m_l} a_{l,k_n}^{(k)} \frac{\partial^{k_n} U_k(x,t)}{\partial x_n^{k_n}} + a_k^{(l)} U_k(x,t) \right] \Bigg|_{x_n=0} = \varphi_l \left(x', t \right),$$

$$(x', t) \in Q_T^{(1)} = Q_T \backslash x_n, m_l \geq 1, l = 1, 2,$$

where Δ is Laplace operator with respect to $x = (x_1, x_2, \ldots, x_n)$; λ_k are given positive constants, and $0 < \lambda_1 < \lambda_2$; $a_{k_n,l}^{(k)}$ $(k = 1, 2)$ are given constants and $\varphi_l \left(x', t \right) \in C_{x',t}^{2,1} \left(Q_T^{(1)} \right)$.

The solution of the boundary problem (1)-(3) is found in the form of a double layer potential. Using the boundary conditions, a system of integro-differential equations (SIDE) is obtained. The characteristic part of the SIDE is solved by the method of Fourier-Laplace integral transforms. The conditions for the correctness and incorrectness of the problem, expressed in terms of the given constants and boundary conditions, are found. Using the regularization method, the SIDE is reduced to a system of Volterra-Fredholm integral equations.

Theorem. If $\varphi_l \left(x', t \right) \in C_{x',t}^{2,1} \left(Q_T^{(1)} \right)$ and $q_k = \frac{\lambda_1 z_k^2 - \lambda_2}{z_k^2 - 1} > 0$ (z_k are roots of the characteristic equation), then the solution of the problem (1)-(3) exists and $U_k(x,t) \in C_{x,t}^{m_k, \left[\frac{m_k}{2} \right]} (Q_T)$.

Keywords: Heat and mass transfer, a boundary value problem, high order normal derivatives, solvability conditions, regularization

2010 Mathematics Subject Classification: 35K45, 58J35

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On well-posedness of source identification elliptic problem with nonlocal boundary conditions

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Abstract: We study the well-posedness of the source identification problem for the two dimensional elliptic differential equation with nonlocal boundary conditions:

(1)
$$
\begin{cases}
-\frac{\partial^2 u(y,x)}{\partial y^2} - a(x)\frac{\partial^2 u(y,x)}{\partial x^2} + \delta u(y,x) = f(y,x) + p(x), \\[2mm]
0 < y < T, 0 < x < l, \\[2mm]
u(0,x) = u(T,x), u_y(0,x) = u_y(T,x),\ u(\lambda,x) = \xi(x), 0 \le x \le l, \\[2mm]
u(y,0) = u(y,l),\ \ u_x(y,0) = u_x(y,l),\ \ 0 \le y \le T,
\end{cases}
$$

where $a(x)$, $\xi(x)$ and $f(y,x)$ are given sufficiently smooth functions and $a(x) > 0,\ 0 < \lambda < T, \delta > 0$ is a sufficiently large number. Assume that all compatibility conditions are satisfied.

Applying operator approaches, the exact estimates for the solution of this problem in Hölder norms are established.

Keywords: Well-posedness, elliptic equations, source identification, exact estimates nonlocal value problem

2010 Mathematics Subject Classification: 35K60, 65M06

A fractional-order two-strain epidemic model with two vaccinations

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Abstract: In this research paper, we extended an existing SIR epidemic integer model containing two strains and two vaccinations by using a system of fractional ordinary differential equations in the sense of Caputo derivative of order $\alpha \in (0,1]$. Four equilibrium points were established: disease free equilibrium, strain1 disease free equilibrium, strain2 disease free equilibrium and endemic equilibrium. Explicit analysis of the equilibrium points of the model was given applying fractional calculus and Routh-Hurwitz criterion. Stability analysis of the equilibrium points was carried out employing the Jacobian matrix. Numerical simulations were iterated to support the analytic results. It was shown that when the two reproduction number R_1 and R_2 are less than one, the disease die out over time and when either of them are greater than one, the disease persist in relation to the thriving strain. The effect of vaccine was also studied. With the fractional order technique, the memory effect of the system is made visible and hence easier to predict.

Keywords: Two strain, vaccine, fractional-order model,basic reproduction number, stability

2010 Mathematics Subject Classification: 34A08, 93A30, 37N25

Estimating Covid-19 deaths by using binomial model

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Abstract: Coronavirus disease 2019, also known as Covid-19, is an infectious disease that has infected more than nineteen million people all around the world. This virus is a member of coronavirus family but it is the most mortal one [1]. It has named as 2019-nCoV by WHO after CDC discovered a new coronavirus from a swab sample of a patient. As we know this pandemic started December 2019 in China, and it is still spreading and causing deaths all around the world [2].

In this paper, we aimed to estimate the right size of epidemic. For that purpose, we chose 10 countries, which are affected by, and still fighting with this disease, to forecast the upcoming death rates by using the previous week deaths. These 10 countries are Argentina, Austria, Brazil, France, Iran, Italy, Sweden, Turkey, United Kingdom, and United States of America. We used the death data of World Health Organization with assumption that data is accurate. For this estimation, firstly, we used the assumption that the reported death delay distributed according to a gamma distribution. Then, we used a binomial distribution for assumption of deaths. This binomial formula led us to find a posterior distribution which is an extension of Bayes' Theorem for death ratio. Lastly, we compared our estimations with real data.

Keywords: Coronavirus disease 2019, SARS-CoV-2, Bayes' Theorem, Posterior Distribution, Binomial Distribution, Gamma Distribution, Deaths.

2010 Mathematics Subject Classification: 46N30, 46L53, 97M10

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The Rayleigh principle for a Sturm-Liouville problem with interior singularity

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Abstract: In this work we are concerned with a Sturm-Liouville problem with interior singularity. By modifying the classical methods we find asymptotic formulas for the eigenvalues and construct the Green's function. Based on Green's function we extend and generalize such spectral properties as Parseval's equality and Rayleigh-Ritz formula. Finally it is established lower bound estimation for the principal eigenvalue.

Keywords: Rayleigh-Ritz formula, Green function, Courants minimax principle

2010 Mathematics Subject Classification: 34B24, 34L10

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The Casimir operator of the group SO(3,2) for parabolic coordinates and related Hamiltonian system

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Abstract: The generators of the quasi-regular representation of the group SO(3,2) are obtained for parabolic coordinates. By the generators, the expression of the quadratic Casimir operator is presented. From the relation between the Casimir operator and Hamiltonian, the corresponding physical system is expressed, and its exact solution is given.

Keywords: Lie group and algebra, Casimir operator, representation, wave function

2010 Mathematics Subject Classification: 22E60, 22E70, 2OC35, 33E15

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The solutions of radial Schrödinger equation by using symmetries

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Abstract: In this study,the solutions of a radial Schrödinger equation which has got the potential $V(r) = 0$ are found by using symmetries. These solutions are compared with the classical solution of the equation.

Throughout this note we mainly use techniques from our works [1,2,3,4,5,6].

Keywords: Symmetry, radial Schrödinger equation

2010 Mathematics Subject Classification: 35J10, 34C14, 76M60

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A numerical algorithm for the involutory hyperbolic problem with the Neumann condition

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Abstracts: In the present paper, the absolute stable first and second order of accuracy difference schemes for the numerical solution of the initial boundary value problem for one dimensional hyperbolic type involutory partial differential equation with Neumann condition are presented. Numerical results are provided.

Keywords: Involutory hyperbolic equation, difference schemes, accuracy

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On the asymptotic formula for the solution of nonlocal boundary value perturbation problems for hyperbolic equations

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Abstract: In the preset paper we consider the nonlocal boundary value perturbation problem

$$\begin{cases} \varepsilon^2 \frac{\partial^2 u(t,x)}{\partial t^2} - (a\,(x)\,u_x\,(t,x))_x + \delta u(t,x) = f(t,x), \\ 0 < t < T, x \in (0,l)\,, \\ u(0,x) = \alpha u(T,x) + \varphi\,(x)\,, x \in [0,l]\,, \\ u^{'}(0,x) = \beta u^{'}(T,x) + \psi\,(x)\,, x \in [0,l]\,, \\ u(t,0) = u(t,l), u_x(t,0) = u_x(t,l), 0 \leq t \leq T, \end{cases}$$

for hyperbolic equation with an arbitrary $\varepsilon \in (0,\infty)$ parameter multiplying the derivative term. An asymptotic formula for the solution of this problem with a small ε parameter is presented.

Keywords: Hyperbolic equations, nonlocal boundary value problem, asymptotic formula.

2010 Mathematics Subject Classification: 35L04, 34B10, 35C20

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Bright soliton solutions for time fractional Korteweg-de Vries (KdV) equation

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Abstract: In this work, He's semi-inverse variation method and the ansatz method with the modified Riemann-Liouville derivative are used to construct exact solutions for time fractional Korteweg-de Vries (KdV) equation. This fractional equation is transformed another non-linear differential equation by travelling wave transformation and then this method is applied to figure out it.

Keywords: Time fractional KdV equation, He's semi-inverse variation method, ansatz method

2010 Mathematics Subject Classification: 26A33, 35R11, 83C15

On the boundedness of solution of the parabolic differential equation with time involution

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Abstract: In the present paper, the initial value problem for the parabolic type involutory partial differential equation is investigated. Applying Green's function of space operator generated by the differential problem, we get formula for solution of this problem. In applications, the theorem on stability estimates for the solution of this problem is established. Furthermore, it is applied to the existence and uniqueness of bounded solution of the nonlinear problem with involution.

Keywords: Hilbert space, Parabolic equation with involution, boundedness

2010 Mathematics Subject Classification: 35J25, 47E05, 34B27

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On the source identification problem for hyperbolic-parabolic equation with nonlocal conditions

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Abstract: Various source identification problems for hyperbolic-parabolic equations and their applications have been investigated in [1]-[3]. In the present paper, a source identification problem for hyperbolic-parabolic equation with nonlocal conditions is studied. Stability estimates for the differential equation of the source identification hyperbolic-parabolic problem are established. Furthermore, stability estimates for the difference scheme of the source identification hyperbolic-parabolic problem are presented.

Keywords: Source identification problem, hyperbolic-parabolic equations, difference scheme, stability estimates.

2010 Mathematics Subject Classification: 65N06, 35M10, 35R30.

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Mobile app evaluation application with AHP method based on interval type-2 fuzzy sets

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Abstract: Nowadays, there are many applications for smart phones that are widely used. These phone applications created initially with small-scale sources. They grow with the investments of developers and appeal to a wider population. However, not every application may have a high level of success. Therefore, which application the investor will support is an important decision making problem. A hierarchy of goal, criteria, sub-criteria and alternatives has been created for this decision making problem. Analytic Hierarchy Process (AHP) [1] method, which has an important place among multi-criteria decision making methods, will be used in the solution. In addition, the group decision making will be applied as it will be beneficial to the result that there will be more than one decision maker. The method will be handled with the Interval Type-2 Fuzzy Sets (IT2FSs) that have high success in handling uncertainties.

Keywords: Multi-criteria decision making, group decision making, AHP method, interval type-2 fuzzy sets, mobile app

2010 Mathematics Subject Classification: 90C70, 90C99

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On the convergence of high-precision finite element method schemes for the two-temperature plasma equation

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Abstract: Mathematical models of physical problems of short-wave oscillations of a two-temperature plasma in an external magnetic field are generally described by the equation [1]

(1)
$$\frac{\partial^2}{\partial t^2}\left(\Delta_3 u - \rho^2 u\right) + \omega^2 \frac{\partial^2}{\partial t^2}\left(\Delta_2 u\right) + \theta^2 \Delta_1 u = f(x,t), \ (x,t) \in Q_T,$$

where $\rho^2, \omega^2, \theta^2 - \text{const} > 0$, depending on Debaevskiy radius or from the Alfven-speed, ω^2 - Langmuir frequency, $\Omega = \{0 \le x_k \le l_k, k = 1,2,3\}$, $Q_T = \{(x,t) : x \in \Omega, t \in (0,T]\}$, $\Delta_3 = \partial^2/\partial x_1^2 + \partial^2/\partial x_2^2 + \partial^2/\partial x_3^2$, $\Delta_2 = \partial^2/\partial x_1^2 + \partial^2/\partial x_2^2$, $\Delta_1 = \partial^2/\partial x_3^2$. Equation (1) is supplemented with the following initial and boundary conditions:

(2)
$$u(x,t)\Big|_{\partial\Omega} = \frac{\partial u}{\partial n}\Big|_{\partial\Omega} = 0, \forall t \in [0,t], \ u(x,0) = u_0(x), \ \frac{\partial u(x,0)}{\partial x} = u_1(x).$$

Approximating the spatial variables in (1) and (2) on the basis of the finite difference method or the finite element method, we obtain a system of ordinary differential equations

(3)
$$D\frac{d^2 u_h(t)}{dt^2} + A u_h(t) = f_h(t), \ u_h(0) = u_{0,h}, \ \frac{du_h}{dt}(0) = u_{1,h}.$$

The operators D, A operate from H_h in H_h. They correspond to the matrix finite element method $D = (a_3(\phi_l, \phi_m))_{l,m-1}^{M}$ and $A = (a_2(\phi_l, \phi_m))_{l,m-1}^{M} + (a_1(\phi_l, \phi_m))_{l,m-1}^{M}$, where $a_m(u,v)$ some bilinear forms. Besides, $u_{k,h} = P_h u_k(x)$, $k = 0,1$, where P_h - design operator $P_h : H \to H_h$.

Further, to solve the problem (3), a multiparametric scheme of the fourth-order finite element method of time accuracy is applied [2]:

(4)
$$D_\gamma \dot{y}_t + A y^{(0.5)} = \Phi_1, \ D_\alpha y_t - D_\beta \dot{y}^{(0.5)} = \Phi_2,$$

(5)
$$y(0) = u_{0,h}, \ \dot{y}(0) = u_{1,h}.$$

Here it is indicated $y = y^n = y(t_n)$, $\dot{y} = \dot{y}^n = \frac{dy}{dt}(t_n)$, $D_\gamma = D - \gamma\tau^2 A$, $D_\beta = D - \beta\tau^2 A$, $y_t = (y^{n+1} - y^n)/\tau$, $\dot{y}_t = (\dot{y}^{n+1} - \dot{y}^n)/\tau$, $y^{(0.5)} = (y^{n+1} + y^n)/2$, $\dot{y}^{(0.5)} = (\dot{y}^{n+1} + \dot{y}^n)/2$, $y^n, \dot{y}^n \in H_h$, $n = 0,1,....$ Further $\Phi_k = \int_0^1 f(t_n + \tau\xi)v_k(\xi)d\xi$, $k = 1,2$, $\xi = (t - t_n)/\tau$, $v_1(\xi) = 1$, $v_2(\xi) = s_1 v_2^{(1)}(\xi) + s_2 v_2^{(2)}(\xi)$, $v_2^{(1)}(\xi) = \tau(\xi - 1/2)$, $v_2^{(2)}(\xi) = \tau(\xi^3 - 3\xi^2/2 + \xi/2)$, $s_1 = 180\beta - 40\alpha$, $s_2 = 1680\beta - 280\alpha$.

A high order of accuracy of the scheme is achieved by special discretization of time and spatial variables. The accuracy in space h^3. The stability and convergence of the constructed algorithms are proved. A priori estimates in various norms are obtained, which are used in the future to obtain consistent estimates of the accuracy of the scheme under weak assumptions about the smoothness of solutions to differential problems. Scheme (4), (5) has certain advantages over other schemes. a) a scheme of high order of accuracy (higher than two); b) in addition to the solution itself, its derivative (speed) is simultaneously found with the same accuracy; c) since the schemes are two-layer, you can use a variable step without loss of accuracy.

Keywords: Finite element method, high order of accuracy, convergence

2010 Mathematics Subject Classification: 65M06, 65M60

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Mathematical modeling of nonlinear problem biological population in not divergent form with absorption, and variable density

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Abstract: Consider the following Cauchy problem for degenerate parabolic equation in not divergence form with absorption and variable density

(1)
$$\frac{\partial u}{\partial t} = u^n \nabla \left(|x|^l \, u^{m-1} \, |\nabla u^k|^{p-2} \, \nabla u \right) + \gamma(t) u - b(t) u^q,$$
$$u(0,x) = u_0(x) \geq 0, x \in R^n$$

Here $u(t,x)$ - the population, numbers n, l, k, p are the given numerical parameters characterizing media, $.0 < \gamma(t), b(t) \in C(0,\infty), q \geq 1$.

The problem (1) in particular value of numerical parameters used for modeling different physical, chemical, biological and other processes [1-4]. To investigating different qualitative properties of the solutions of the problem Cauchy (1) and boundary value problem for particular value of numerical parameters devoted many works. For instance, in the case $l = n = 0, m = k, 0 < q < 1, \gamma(t) = 0$ by analyzing an exact solution [2] establish the following properties of solutions: an inertial effect of a finite velocity of propagation of thermal disturbances, spatial heat localization and finite time localization solution effect. The problem (1) when $l = n = 0, m = k = 1, q \geq 1, \gamma(t) = b(t) = const$ is well-known equation of biological population studied in [4].

In this work it is find exact solution and based it the following properties an effect of a finite velocity population flash disturbances, spatial localization population flash established. It is obtained estimate of weak solution and free boundary, the condition of Fujita type global solvability considered problem proved. The results of the numerical experience discussed.

Keywords: Mathematical modeling, nonlinear problem, biological population, not divergent, absorption, variable density

2010 Mathematics Subject Classification: 65N06,35M10, 35R30

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Interpolation of data in $\mathbb{R}^3$ using quartic triangular Bézier surfaces

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Abstract: We consider the problem of interpolation of scattered data in $\mathbb{R}^3$ and propose a solution based on Nielson's minimum norm network and triangular Bézier patches. Our algorithm applies splitting to all triangles of an associated triangulation and constructs G^1-continuous bivariate interpolant consisting of quartic triangular Bézier patches. The algorithm is computationally simple and produces visually pleasant smooth surfaces. We have created a software package for implementation, 3D visualization and comparison of our algorithm and the known Shirman and Séquin's method which is also based on splitting and quartic triangular Bézier patches. The results of our numerical experiments are presented and analysed.

Keywords: scattered data interpolation, minimum norm networks, triangular Bézier surfaces

2010 Mathematics Subject Classification: 65D05, 65D17, 68U05

A new numerical algoritm for solving Sturm-Liouville problems with discontinuities in an interior point

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Abstract: Boundary value problems for Sturm-Liouville type equations are used to understand various type of physical problems in nature. To understand how nature works, we need to solve these Sturm-Liouville problems using some analytical methods. [1] In most cases, it is impossible or difficult to solve these modeled problems under realistic boundary conditions analytically. So we prefer different numerical methods to solve these problems. [3] The numerical methods, such as differential transform method, the shooting method, the homotopy pertubation method, the Adomian decomposition method, finite difference method and etc. are powerful approximate methods for solving various kinds of initial and/or boundary value problems. The main advantage of finite difference method (FDM) is that it can be applied efficiently to a rather wide class of boundary value problems. [2] In this work, we will adapt the FDM to discontinuous Sturm-Liouville problems the main feature of which is the nature of the equations and the boundary conditions imposed. Namely, the boundary conditions contains not only boundary points of the considered interval, but also an interior point of discontinuity at which given additional transmission conditions, so our problem is the nonclassical once. Based on Finite Difference method and our own approaches a new numerical algoritm is introduced for such type transmission problems.

We consider Sturm-Liouville equation,

$$(1) \qquad y^{''}(x) + p(x)y^{'}(x) + q(x)y(x) = f(x), \quad x \in [a,b]$$

subject to separate boundary conditions

$$(2) \qquad y(a) = \alpha, \quad y(b) = \beta$$

with additional transmission conditions at the interior point $c \in (a,b)$, given by

$$(3) \qquad y(-c) = my(+c) \qquad y'(-c) = ny'(+c)$$

where p(x), q(x) and f(x) are continuous functions on [a,b] and α, β m,n are real constants,$m \neq 0, n \neq 0$.

It is our main goal here to develop FDM to deal with the discontinuous boundary-value problems involving additional transmission conditions at the the point of discontinuity. By comparision with the exact solutions we show that the our own numerical algoritm, which based on FDM is an efficient method for solving Sturm-Liouville type problems under supplementary transmission conditions.

2010 Mathematics Subject Classification: 34A36, 34B09, 65L10, 65L12

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Using the resources of a distributed information system for solving applied problems of hydrodynamics

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Abstract: High performance hardware is developing very rapidly with the advances in industrial productions. It contributes to the introduction of numerical methods [1]. Possibilities of solving labor-consuming mathematical problems using distributed computing are considered. Distributed computing is a way to solve time-consuming computational problems using several computers, most often combined into a parallel computing system. Distributed computing is also applicable in distributed control systems. The created distributed information system fully provides the necessary computing resources for current research and educational processes simplifying the prospects for its further development, and allows you to create a developed IT infrastructure for managing intellectual property, an electronic library designed to store all books and scientific works of the Kazakhstan Engineering and Technological University and research institutes of Almaty [2-3]. On the basis of the created distributed information system, further on the portal https://www.acagor.kz/, it is possible not only to store digitized works of scientists from research institutes in Almaty, but also to use the resources of this portal to solve applied mathematical problems. For a numerical solution using the portal resources, the following industrial applications were considered. One of these tasks is to evaluate the efficiency and predict the indicators of oil field development. Due to the complexity of the mathematical models describing these processes, calculations for one field can last from several hours to several days. Therefore, the issue of developing efficient parallel algorithms based on distributed computing systems that can significantly speed up computations becomes relevant. Along with the classical filtration model based on Darcy's law, a number of other models are widely used in the study of fluid flows in oil reservoirs, such as the models of Zhukovsky N.E. [4], Forchheimer [5] and Navier-Stokes [4, 5]. The use of these models is associated with violation of Darcy's law under certain conditions, the need for a detailed study of filtration processes near wells, etc. The second problem considered is the numerical simulation of convective flows based on the Navier-Stokes equations. Initially a stable explicit difference scheme is constructed for the numerical solution of the Navier-Stokes equations in a multiple connected domain using the pressure uniqueness condition and the fictitious domain method for solving this problem. To solve this problem, an alternative approximate method is considered based on the method of fictitious domains with continuation by lower coefficients. The discretization of the obtained equations in the work is carried out by the method of finite differences. The distributed information system has the following functionality:

- Providing efficient high-performance computing for solving problems of numerical modeling;
- Reliable storage and management of large data, and the results of numerical simulation;
- Organization of work with the possibility of remote access.

Using installed servers on the basis of the created integrated distributed information system, it is possible to numerically solve time-consuming mathematical problems. These are very good opportunities for a researcher who does not have his own resources to solve any labor-intensive tasks.

Keywords: IT infrastructure, cloud solutions, data center, server, distributed computing systems, hydrodynamics, numerical solution

2010 Mathematics Subject Classification: 65Q10, 65K05

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A note on hyperbolic differential equations on manifold

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Abstract: The well-posedness of nonlocal boundary value problems for partial differential equations of hyperbolic type has been studied extensively (see, e.g. [1, 2, 2, 4] and the references therein).

The present abstract investigates the differential equations on smooth closed manifolds and considers the well-posedness of boundary value problem for hyperbolic equations in Hölder spaces. Moreover, it is established new coercivity estimates in various Hölder norms for the solutions of such boundary value problems for hyperbolic equations.

Keywords: Differential equations on manifolds, well-posedness, self-adjoint positive definite operator

2010 Mathematics Subject Classification: 58Jxx, 58J32, 58J99

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Numerical solution to system of linear integro differential difference equations using polynomial collocation method

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Abstract: In this research, we consider polynomial collocation method for the solution of linear integro differential difference equations with mixed conditions. The uniqueness and convergence of the solution are established. The state equation is transformed into system of linear algebraic equations using standard collocation points and then solved using matrix inversion method. The solution of the algebraic equations is substituted into the approximate solution to obtain the numerical solution. The method is shown to be convergent, numerical examples show that the method is efficient in terms of consistency and accuracy.

Keywords: Operational matrix, integro differential equation, difference equation, mixed condition, Fredholm integro differential equation

Numerical solution for the second order of accuracy difference scheme for the source identification elliptic-telegraph problem

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Abstract: Identification problems take an important place in applied sciences and engineering applications and have been studied by many authors (see, e.g., [1]-[4] and the references given therein).
In the present paper, source identification problem for the elliptic-telegraph equation is investigated. The second order of accuracy absolute stable difference scheme for the numerical solution of the one dimensional identification problem for the elliptic-telegraph equation with the Dirichlet condition is presented. Some numerical results are provided.

Keywords: Source identification problem, elliptic-telegraph equation, difference schemes

2010 Mathematics Subject Classification: 35J25, 47E05, 34B27

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A note on mathematical theory of epidemics: SIR modeling of the COVID-19

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Abstract: A considerable amount of research works has been devoted for the epidemic outbreak caused by coronavirus COVID-19. The high rate of the infection spread and the number of fatalities makes the understanding of the current epidemiological models more important than ever before. The most relevant mathematical models relating to the spread of a pandemic is the susceptible-infectious-removed (SIR) [3], [6] model, susceptible-exposed-infectious-removed (SEIR) [4], [3], [7] model, the susceptible - infectious - susceptible (SIS) [1], [2] model, the susceptible-unquarantined-quarantined-confirmed (SUQC) [9] model. For long-time predictions, more complicated mathematical models are necessary which makes the procedure difficult if reliable data are limited as the complicated models need more effort to calculate unknown parameters. In this study, we consider well known SIR model which is proposed by McKendrick and Kermack [8] by the following nonlinear system of ordinary differential equations.

$$(4) \qquad \begin{cases} \dfrac{dS}{dt} = -\beta I S \\[2mm] \dfrac{dI}{dt} = \beta I S - \gamma I \\[2mm] \dfrac{dR}{dt} = -\gamma I \end{cases}$$

In this model a fixed population with only three compartments is considered: susceptible (S), infected (I), and recovered (R), respectively. The unknowns used for this model consist of three classes:

a) $S(t)$ represents the number of individuals not yet infected with disease at time t or those susceptible to the disease,

b) $I(t)$ denotes the number of individuals who have been infected with the disease, and are capable of spreading the disease to those in the susceptible category, and

c) $R(t)$ is the compartment of the individuals who have been infected and then recovered from the disease.

There are various approaches to understand the predictions of this model and the behavior of its solutions. Kermack and McKendrick [8] reduced this problem to a single differential equation and derived an approximate solution for the removal rate, dR/dt, in terms of a hyperbolic secant function by neglegting some terms. In this study, we obtain an approximate solution by taking into consideration the term neglegted in Kermack and McKendrick [8]. The results of the numerical experiments has similar features with the well-known strategies which shows that it can be used to forecast COVID-19 epidemic situation. This present work will be of great support to the epidemiology to investigate the validity of these models which is already being considered and used by different researchers.

Keywords: COVID-19, susceptible, infectious, recovered, mathematical model, SIR

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Numerical solution for the source identification telegraph problem with Neumann condition

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Abstract: The source identification problem for the telegraph equation is studied. We propose The first order of accuracy absolute stable difference scheme for the numerical solution of the one dimensional identification problem for the telegraph equation with the Neumann condition . Computational efficiency of the method is confirmed through example whose results are in good agreement with theory. The obtained numerical results have been compared with the exact solution to verify the accurate nature of our method.

Keywords: Source identification problem, telegraph equation, difference schemes

2010 Mathematics Subject Classification: 35J25, 47E05, 34B27

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A solvability conditions of 2-d nonlocal boundary value problem for Poisson's operator on rectangle

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We report that new accurate conditions of solvability for the differential and difference problems [1] are established, i.e., next theorems are proved.

<u>Theorem 1.</u> Let $\Pi = (0 < x < 1) \times (0 < y < \pi)$ and $f \in C(\overline{\Pi})$. If

$$\sum_{r=1}^{n} \alpha_r - \sum_{s=1}^{m} \beta_s < \frac{\sinh 1}{\sinh \zeta_n} \quad for \quad \zeta_n < \eta_1, \quad or \quad \sum_{r=1}^{n} \alpha_r < \frac{\sinh 1}{\sinh \zeta_n} \quad for \quad \zeta_n > \eta_1,$$

then, belonged under $C(\overline{\Pi}) \cap C^2(\Pi)$, classical solution u of the problem

$$(1) \quad \begin{cases} \Delta u(x,y) = f(x,y), \ (x,y) \in \Pi; \ u(x,0) = u(x,\pi) = 0, \ 0 \le x < 1, \\ u(0,y) = 0, \ u(1,y) = \sum_{r=1}^{n} \alpha_r u(\zeta_r, y) - \sum_{s=1}^{m} \beta_s u(\eta_s, y), \ 0 \le y \le \pi, \\ 0 < \zeta_1 < ... < \zeta_n < 1, \ 0 < \eta_1 < ... < \eta_m < 1, \\ \zeta_r \ne \eta_s, \ \alpha_r > 0, \ \beta_s > 0, \ r = \overline{1,n}, \ s = \overline{1,m} \end{cases}$$

exists, it is an a unique and a priori estimate $||u||_{W_2^2(\Pi)} \le C||f||_{L_2(\Pi)}$ holds.

<u>Theorem 2.</u> Let θ is less than a half of a distance between any two points $0, \zeta_1, ..., \zeta_n, \eta_1, ..., \eta_m, 1$. Let $u \in C^4(\overline{\Pi})$ is the solution of (1) when

$$\sum_{r=1}^{n} \alpha_r - \sum_{s=1}^{m} \beta_s < \left(1 + \frac{4}{\pi}\right)^{1-\zeta_n-\theta} \ if \ \zeta_n < \eta_1, \ or \ \sum_{r=1}^{n} \alpha_r < \left(1 + \frac{4}{\pi}\right)^{1-\zeta_n-\theta} \ if \ \zeta_n > \eta_1,$$

then mesh solution Y of the difference scheme (3), which is proposed in [1, p. 070021-3], approximates $u(x,y)$ by the second order of accuracy in terms of $h = \sqrt{h_1^2 + h_2^2}$ if $h_2 \to 0$ in each of the difference metrics C and W_2^2.

Keywords: 2-d nonlocal boundary value problem, Poisson's operator

2010 Mathematics Subject Classification: 35B45, 35J05, 65N06

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On some differential inequalities for first order Fuzzy differential equations

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Abstract: In this paper, several comparison results for first order fuzzy differential equations are proved via tecnique of upper and lower solutions. Also, some comparison results are evidenced under initial time difference. Hukuhara derivative, Hukuhara difference and partial orderings are used to prove theorems. Besides, applicability of this comparison results are shown by giving an example.

Keywords: Fuzzy upper and lower solutions, Fuzzy differential equations, Hukuhara difference, Hukuhara derivative, partial ordering

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On the stability of solution of the parabolic differential equation with time involution

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Abstract: The initial value problem for the parabolic type involutory partial differential equation is studied. Applying Green's function of space operator generated by the differential problem, we get formula for solution of this problem. The theorem on stability estimates for the solution of this problem with involution is established

Keywords: Hilbert space, parabolic equation with involution, stability.

2010 Mathematics Subject Classification: 35J25, 47E05, 34B27

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On the stability of second order of accuracy difference scheme for the numerical solution of the time delay telegraph equation

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Abstract: In this study, second order of accuracy difference scheme for approximate solution of initial-boundary value problem for time delay telegraph equation with Dirichlet condition is presented. The main theorem on stability of the difference scheme is established. Numerical results are provided.

Keywords: Difference schemes, stability, delay telegraph equations

2010 Mathematics Subject Classification: 65M06, 35L10, 35L20

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Implicit method of second order accuracy on hexagonal grids for approximating the first derivatives of the solution to heat equation on a rectangle

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Abstract: The Dirichlet type boundary value problem of heat equation

$$(1) \qquad \frac{\partial u}{\partial t} = \omega \left(\frac{\partial^2 u}{\partial x_1^2} + \frac{\partial^2 u}{\partial x_2^2} \right) + f(x_1, x_2, t)$$

on a rectangle, where $\omega > 0$, is constant and f is the heat source is considered. We propose a two stage implicit method for the approximation of the first order derivatives of the solution $u(x_1, x_2, t)$ with respect to the spatial variables x_1 and x_2. At the first stage a two layer implicit method on hexagonal grids with order of accuracy $O(h^2 + \tau^2)$ given in [1] is used to approximate the solution $u(x_1, x_2, t)$. At the second stage we propose special difference boundary value problems on hexagonal grids for the approximation of $\frac{\partial u}{\partial x_1}$ and $\frac{\partial u}{\partial x_2}$ of which the boundary conditions are defined by using the obtained solution from the first stage. It is proved that the given implicit schemes of the special difference boundary value problems are unconditionally stable. It is also showed that the solution of these difference boundary value problems converge to the corresponding exact derivatives $\frac{\partial u}{\partial x_1}$ and $\frac{\partial u}{\partial x_2}$ on the grids of order $O(h^2 + \tau^2)$ where, h and $\frac{\sqrt{3}}{2}h$ are the step sizes in space variables x_1 and x_2 respectively and τ is the step size in time. The method is applied on test problems and the obtained numerical results justify the given theoretical results.

Keywords: Finite difference method, hexagonal grid, stability analysis, two dimensional heat equation, approximation of derivatives

2010 Mathematics Subject Classification: 65M06, 65M12, 65M22

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On generalization of Hermite-Hadamard inequality for bivariate log-convex stochastic processes

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Abstract: In the paper, the author defines a stochastic process $S : \Xi \times \Sigma \to \mathbb{R}_+$ that called "bivariate log-convex" on $\Xi \subseteq \mathbb{R}^2$. Then, the author establishes the Hermite-Hadamard type integral inequalitiy for these processes, and generalizes classical Hermite-Hadamard inequality for integrals to these processes as follows:

$$\frac{\delta_2 - \theta_2}{2n} \sum_{i=1}^{n} \int_{\theta_1}^{\delta_1} S((\omega, A(\rho_{i-1}, \rho_i)), \cdot)\mathrm{d}\omega$$

$$+\frac{\delta_1 - \theta_1}{2n} \sum_{i=1}^{n} \int_{\theta_2}^{\delta_2} S((A(\omega_{i-1}, \omega_i), \rho), \cdot)\mathrm{d}\rho \leq \int_{\theta_1}^{\delta_1} \int_{\theta_2}^{\delta_2} S((\omega, \rho), \cdot)\mathrm{d}\omega\mathrm{d}\rho$$

$$\leq \quad \frac{\delta_2 - \theta_2}{2n} \sum_{i=1}^{n} \int_{\theta_1}^{\delta_1} L\left(S((\omega, \rho_{k-1}), \cdot), S((\omega, \rho_k), \cdot)\right)\mathrm{d}\omega$$

$$+\frac{\delta_1 - \theta_1}{2n} \sum_{i=1}^{n} \int_{\theta_2}^{\delta_2} L\left(S((\omega_{k-1}, \rho), \cdot), S((\omega_k, \rho), \cdot)\right)\mathrm{d}\rho,$$

where $\omega_i = \theta_1 + i\frac{\delta_1 - \theta_1}{n}$, $\rho_i = \theta_2 + i\frac{\delta_2 - \theta_2}{n}$, $i = 0, 1, 2, \ldots, n$; and $n \in N$.

Keywords: Log-convexity, bivariate stochastic process, mean-square integral, Hermite-Hadamard inequality

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Several fractional integral inequalities for symmetrized convex stochastic processes

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Abstract: In this study, a stochastic process $S : [\theta, \delta] \times \Sigma \to \mathbb{R}$ $(\theta < \delta)$ is introduced symmetrized convex on the interval $[\theta, \delta]$, if the symmetrical stochastic transform $\widetilde{S}$ which is defined by

$$(1) \qquad \widetilde{S} := \frac{1}{2} \left[S(\omega, \cdot) + S(\theta + \delta - \omega, \cdot) \right], \omega \in [\theta, \delta]$$

is convex on $[\theta, \delta]$. Then, the authors obtain the Hermite-Hadamard type inequalitiy via fractional integral operators for these processes, as follows:

$$(2) \qquad S\left(\frac{\theta + \delta}{2}, \cdot\right) \leq \frac{\Gamma(\alpha + 1)}{2(\delta - \theta)^{\alpha}} \left[I_{\theta+}^{\alpha} S(\delta, \cdot) + I_{\delta-}^{\alpha} S(\theta, \cdot) \right] \leq \frac{S(\theta, \cdot) + S(\delta, \cdot)}{2}.$$

Moreover, the related results of the above inequality is verified in this study.

Keywords: Symmetric-convexity, stochastic process, mean-square integral, Hermite-Hadamard inequality, fractional intagral operator

2010 Mathematics Subject Classification: 26D15, 26D99, 26A51, 46N10

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Some quantum integral inequalities for convex stochastic processes

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Abstract: In this study, the authors obtain the following q-Hermite-Hadamard type inequalities for convex mean-square differentiable stochastic process $S : [\theta, \delta] \times \Sigma \to \mathbb{R}$ on $[\theta, \delta]$ and $0 < q < 1$ as follows:

$$(1) \qquad S\left(\frac{q\theta + \delta}{1 + q}, \cdot\right) \leq \frac{1}{\delta - \theta} \int_{\theta}^{\delta} S\left(\omega, \cdot\right) \ _{\theta}d_{q}\omega \leq \frac{qS\left(\theta, \cdot\right) + S\left(\delta, \cdot\right)}{1 + q};$$

$$
\begin{aligned}
& S\left(\frac{\theta + \delta}{2}, \cdot\right) + \frac{(1 - q)(\delta - \theta)}{2(1 + q)} S'\left(\frac{\theta + \delta}{2}, \cdot\right) \\
(2) \qquad & \leq \quad \frac{1}{\delta - \theta} \int_{\theta}^{\delta} S\left(\omega, \cdot\right) \ _{\theta}d_{q}\omega \leq \frac{qS\left(\theta, \cdot\right) + S\left(\delta, \cdot\right)}{1 + q};
\end{aligned}
$$

$$
\begin{aligned}
& S\left(\frac{\theta + q\delta}{1 + q}, \cdot\right) + \frac{(1 - q)(\delta - \theta)}{1 + q} S'\left(\frac{\theta + q\delta}{1 + q}, \cdot\right) \\
(3) \qquad & \leq \quad \frac{1}{\delta - \theta} \int_{\theta}^{\delta} S\left(\omega, \cdot\right) \ _{\theta}d_{q}\omega \leq \frac{qS\left(\theta, \cdot\right) + S\left(\delta, \cdot\right)}{1 + q}.
\end{aligned}
$$

Then, the quantum estimates for midpoint type inequalities thank to the above results are verified in this study.

Keywords: Convex stochastic process, mean-square differentiable, q-Hermite-Hadamard inequality

2010 Mathematics Subject Classification: 26D15, 26D99, 26A51, 46N10

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Local and global dynamics of a cancer tumor growth model with multipoint structure

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Abstract: We present a phase-space analysis of a mathematical model of tumor growth with an immune responses. We consider mathematical analysis of the model equations with multipoint initial condition regarding to dissipativity, boundedness of solutions, invariance of non-negativity, local and global stability and the basins of attractions. We derive some features of behavior of the three-dimensional tumor growth models with dynamics described in terms of densities of three cells populations: tumor cells, healthy host cells and effector immune cells. We found sufficient conditions, under which trajectories from the positive domain of feasible multipoint initial conditions tend to one of equilibrium points. Here, cases of the small tumor mass equilibria-the healthy equilibrium point, the "death" equilibria have been examined. Biological implications of our results are discussed. In contrast to mentioned works, here mathematical analysis of multipoint IVP for local and global stability and the multiphase basins of attractions have been investigated. Here, we consider the following multipoint initial value problem (IVP) for dynamical system

$$\dot{T} = r_1 T \left(1 - k_1^{-1} T \right) - a_{12} NT - a_{13} TI,$$

(1)
$$\dot{N} = r_2 N \left(1 - k_2^{-1} N \right) - a_{21} NT, \ \dot{I} = \frac{r_3 IT}{k_3 + T} - a_{31} IT - d_3 I,$$

(2)
$$T(t_0) = T_0 + \sum_{k=1}^{m} \alpha_{1k} T(t_k), \ N(t_0) = N_0 + \sum_{k=1}^{m} \alpha_{2k} N(t_k),$$

$$I(t_0) = I_0 + \sum_{k=1}^{m} \alpha_{3k} I(t_k), \ t_0 \in [0, \ \eta), \ t_k \in O_\delta(t_0),$$

where $T = T(t)$, $N = N(t)$, $I = I(t)$ denote the densities of tumor cells, healthy host cells and the effector immune cells, respectively at the moment t, m is a natural number and α_{ij} are real numbers.

The assumption (2) is given on coefficients α_{ij} and $t_0, t_1, t_2, ...t_m$. Here, (T_0, N_0, I_0) indicate the given pre-healing vector (or pre-healing vector state) such that T_0 is small enough but N_0, I_0 are big enough. The condition (2) links the values of vector function $V(t) = (T(t), N(t), I(t))$ at various points $t_0, t_1, ..., t_m$ with each other by healing vector (T_0, N_0, I_0). So, we called (2) a multipoint IVP.

Michaelis-Menten dynamics of a cancer tumor growth model with multiphase structure

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Abstract: We studied a phase-space analysis of a mathematical model of tumor growth with an immune responses. The addition of a drug term to the system can move the solution trajectory into a desirable basin of attraction. We show that the solutions of the model with a time-varying drug term approach can be evaluated by a more fruitful way in down to earth style. We studied some features of behavior of one of three-dimensional tumor growth models with dynamics described in terms of densities of three cells populations: tumor cells, healthy host cells and effector immune cells. In this paper, we examine the dynamics of one cancer growth model proposed in [1], but possessing multiphase structure, i.e. we consider the dynamical system

$$(1) \qquad \dot{E}(t) = cT - \mu_2 E + \frac{p_1 EI}{\alpha_1 + I} + s_1 - g_1(u) E,$$

$$\dot{T}(t) = r_2(T) T - \frac{aET}{\alpha_2 + T} - g_2(u) T,$$

$$\dot{I}(t) = \frac{p_2 ET}{\alpha_3 + T} - \mu_3 I + s_2 - g_3(u) I, \ \dot{u} + d_2 u(t) = \upsilon(t)$$

with multipoint initial condition

$$(2) \qquad E(t_0) = E_0 + \sum_{k=1}^{m} \alpha_{2k} E(t_k), \ T(t_0) = T_0 + \sum_{k=1}^{m} \alpha_{1k} T(t_k),$$

$$I(t_0) = I_0 + \sum_{k=1}^{m} \alpha_{3k} I(t_k), \ t_0 \in [0, \ \delta), \ t_k \in (0, \delta), \ t_k > t_0,$$

where $E = E(t)$, $T = T(t)$, $I = I(t)$ denote the densities of effector-cell, tumor cells and the concentration of interleukin-2 (IL-2) cells, respectively, at the moment t, $u(t)$ denotes the amount of drug at the tumor site at time t, this is determined by the dose given $\upsilon(t)$.

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The non-commutative Hardy-Littlewood maximal function on symmetric spaces of $\tau-$measurable operators

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Abstract: In this paper, we investigate the Hardy-Littlewood maximal function on non-commutative symmetric spaces. We complete the results of T. Bekjan and J. Shao. Moreover, we refine the main results of the papers [1] and [2].

Keywords: Symmetric spaces of functions and operators, Hardy-Littlewood maximal function, von Neumann algebra, (non-commutative) Lorentz and Marcinkiewicz spaces

2010 Mathematics Subject Classification: 46E30, 47B10, 46L51, 46L52, 44A15, 47L20, 47C15

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On the periodicity of solutions of a system of rational difference schemes

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Abstract: In this paper, we have investigated the periodicity of the well-defined solutions of the system of difference equations:

(1) $$u_{n+1} \equiv -\frac{u_{n-1}+v_n}{\alpha v_n u_{n-1}-1}, \ \ v_{n+1} \equiv -\frac{v_{n-1}+u_n}{\alpha u_n v_{n-1}-1}, \ \ w_{n+1} \equiv -\frac{u_n}{v_n}$$

where

(2) $$u_0, u_{-1}, v_0, v_{-1}, w_0, w_{-1} \in \mathbb{R}\backslash\{0\} \ \text{ and } \ \alpha > 0.$$

Note that system (1) can be written as system

(3) $$x_{n+1} \equiv -\frac{x_{n-1}+y_n}{y_n x_{n-1}-1}, \ \ y_{n+1} \equiv -\frac{y_{n-1}+x_n}{x_n y_{n-1}-1}, \ \ z_{n+1} \equiv -\frac{x_n}{y_n}$$

by the change of variables ,

(4) $$u_n \equiv -\frac{x_n}{\sqrt{\alpha}}, \ \ v_n \equiv -\frac{y_n}{\sqrt{\alpha}}, \ \ w_n \equiv -z_n.$$

That's why, we will consider system (4) instead of system (1) for the remaining part of the paper.

Main Result.

Theorem 2.1. *Let $y_0 = a$, $y_{-1} = b$, $x_0 = c$, $x_{-1} = d$, $z_0 = e$, $z_{-1} = f$ be nonzero arbitrary real numbers and $\{x_n, y_n, z_n\}$ be a solution of system (3). Also, assume that $ad \neq 1$, $bc \neq 1$ and $(b+c) \neq 0$ and $(d+a) \neq 0$. Then, all solutions of system (3) are as following:*

(5) $$x_n = \begin{cases} \frac{d+a}{ad-1}, & n = 6k+1 \\ b, & n = 6k+2 \\ a, & n = 6k+3 \\ \frac{b+c}{cb-1}, & n = 6k+4 \\ d, & n = 6k+5 \\ c, & n = 6k+6 \end{cases} \quad for \ k \in \mathbb{N}_0$$

(6) $$y_n = \begin{cases} \frac{b+c}{cb-1}, & n = 6k+1 \\ d, & n = 6k+2 \\ c, & n = 6k+3 \\ \frac{d+a}{ad-1}, & n = 6k+4 \\ b, & n = 6k+5 \\ a, & n = 6k+6 \end{cases} \quad for \ k \in \mathbb{N}_0$$

(7) $$z_n = \begin{cases} \frac{c}{a}, & n = 6k+1 \\ \frac{(d+a)(cb-1)}{(ad-1)(b+c)}, & n = 6k+2 \\ \frac{b}{d}, & n = 6k+3 \\ \frac{a}{c}, & n = 6k+4 \\ \frac{(b+c)(ad-1)}{(cb-1)(d+a)}, & n = 6k+5 \\ \frac{d}{b}, & n = 6k+6 \end{cases} \quad for \ k \in \mathbb{N}_0$$

Proof. We prove the theorem by induction for k. ☐

Keywords: Difference equation, system; solutions, periodicity

2010 Mathematics Subject Classification: 35J05, 35J08, 35J25

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Mathematical Modeling for Course of COVID-19 Pandemics in Libya

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Abstract: The disease known as COVID-19, caused by a new type of coronavirus, first appeared in China at the end of 2019. The first COVID-19 case in Libya was seen in the middle of February 2020, and 7738 cases have been reached to August 15 2020. The number of deaths caused by COVID-19 in Libya has been 145 until same time [1]. In this study, it was aimed to determine the course of COVID-19 infection in Libya with a new modified mathematical modeling [2,3] and to show the possible number of cases and deaths in the upcoming period. The results obtained separately on a total and daily basis were shown graphically.

Keywords: COVID-19, mathematical modeling, coronavirus pandemics, Libya

2010 Mathematics Subject Classification: 93A30, 90B06

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New NLBVP with Poisson's operator in rectangle

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Abstract: We established a priori estimate of classical solution for new NLBVPs. Let

(1)
$$\begin{cases} \Delta u(x,y) = f(x,y), \ (x,y) \in \Pi, \\ u(x,0) = u(x,\pi) = 0, \ 0 \le x < 1, u(0,y) = 0, \ 0 \le y \le \pi, \end{cases}$$

(2)
$$u_x(1,y) = \sum_{r=1}^{n} \alpha_r u_x(\zeta_r, y) - \sum_{s=1}^{m} \beta_s u_x(\eta_s, y), \ 0 \le y \le \pi,$$

(3)
$$u_x(1,y) = \int_0^{\tau_1} \rho(x) u_x(x,y) dx, \ 0 \le y \le \pi,$$

$\Pi = (0,1) \times (0,\pi)$, $f \in C(\overline{\Pi})$, $\alpha_r > 0$, $\beta_s > 0$, $\zeta_r \neq \eta_s$ $r = \overline{1,n}$, $s = \overline{1,m}$, $0 < \zeta_1 < ... < \zeta_n < 1$,
$0 < \eta_1 < ... < \eta_m < 1$, $\rho \in C[0,\tau_1]$, $\tau_1 \le 1$. Let solution of (1)-(2) or (1)-(3) $u \in C^2(\Pi) \cap C(\overline{\Pi} \setminus \partial\Pi|_{x=1})$, $u_x \in C(\partial\Pi|_{x=1})$.

<u>Theorem 1.</u> If $\sum_{r=1}^{n} \alpha_r - \sum_{s=1}^{m} \beta_s < \frac{\cosh 1}{\cosh \zeta_n}$ for $\zeta_n < \eta_1$, $\sum_{r=1}^{n} \alpha_r < \frac{\cosh 1}{\cosh \zeta_n}$ for $\zeta_n > \eta_1$, then a solution of (1)-(2) holds a priori estimate

(4)
$$||u||_{W_2^2(\Pi)} \le C||f||_{L_2(\Pi)}.$$

<u>Theorem 2.</u> If (a) $\int_0^{\tau_1} \rho(x) dx < \frac{\cosh 1}{\cosh \tau_1}$ and $\rho(x)$ does not change a sign, or

(b) $\int_0^{\tau_1} \rho(x) dx < \frac{\cosh 1}{\cosh \tau_0}$ and the sign is changing from plus to minus in $\tau_0 \in (0,\tau_1)$, or

$\int_0^{\tau_1} \frac{\rho(x) + |\rho(x)|}{2} dx < \frac{\cosh 1}{\cosh \tau_1}$ and behaviour of $\rho(x)$ differs from (a)-(b), then a solution of (1),(3) holds a priori estimate (4).

Keywords: a priori estimate, NLBVP, Poisson's operator, rectangle

2010 Mathematics Subject Classification: 35B45, 35J05

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On the boundedness of solution of the second order ordinary differential equation with dumping term and involution

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Abstract: In the present paper, the initial value problem for the second order ordinary differential equation with dumping term and involution is investigated. We establish equivalent initial value problem for the fourth order ordinary differential equations to the initial value problem for second order linear differential equations with dumping term and involution. Theorem on stability estimates for the solution of the initial value problem for the second order ordinary linear differential equation with dumping term and involution is proved. Theorem on existence and uniqueness of bounded solution of initial value problem for second order ordinary nonlinear differential equation with dumping term and involution is established.

Keywords: Involution, boundedness, existence and uniqueness

2010 Mathematics Subject Classification: 35J25, 47E05, 34B27

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Mathematical modelling of covid-19 with the effect of vaccine

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Abstract: Covid-19 is the most recently discovered infectious disease affecting the countries all around the world. SARS-CoV-2, which is a member of coronavirus family, is the virus that makes the infection. Until the 28th of September 2020, almost 34 million people infected by the virus and more than 1 million people died all around the world [1].

One of the most discussed ideas about the disease to die out is vaccination [2]. In our study, we tried to analyze this idea and show the effect of vaccine for Covid-19. Our work starts with constructing an SIV mathematical model. Afterwards, we made the analyze of our model. Then, by taking into consideration of incoming passengers and precautions that should be taken, we used the vaccination idea with changing the percentage of vaccinated people in a population. In last section, we used numerical simulations to support our idea. In our work, we conclude that vaccination is substantially effective if we consider the other things that affect the disease such as incoming passengers and precautions like wearing mask, maintaining social distance, etc.

Keywords: Covid-19, mathematical modelling, vaccine, pandemic.

2010 Mathematics Subject Classification: 46N30, 46L53, 97M10

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Sensitivity analysis on the SEIR-SEI model for the dynamics of blinding trachoma

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Abstract: Trachoma was one of 17 preventable diseases (NTDs) that the World Health Organization (WHO) prioritized for surveillance and elimination through protective therapies or improved prevention and treatment approaches [1]. In many engineering and science fields sensitivity analysis has become highly interesting. For the mathematical modeling of biological phenomena, researchers use sensitivity and uncertainty analysis because of its usefulness for defining important parameters for performance of the model. It can also help in the process of experimental analysis, reducing model order, estimating parameters, taking decisions or developing recommendations for decision-makers. Here, we illustrated the use of the local sensitivity analysis to explain the effect of various parameters on a threshold parameter, R_0, resulting from the study of a dynamics model inside the human-host. And it is confirmed from computed elasticity indices that the most sensitive parameter to basic reproduction number is (vector contact rate) followed by rates of transmission. Moreover, a detailed parameter estimation of the model parameters and model fitting presented with the use of field data cases from Northern Nigeria using least-square fitting method. Finally, the sensitivity analysis results shows that improving the rate of environmental hygiene and facial cleanliness will attract a consequential decrease in the size of basic reproduction number, which results in the declination of the disease transmission.

Keywords: Trachoma, mathematical modelling, sensitivity analysis, basic reproduction number.

2010 Mathematics Subject Classification: 46N30, 46L53, 97M10

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Effective reproduction number for North Cyprus fighting covid-19

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Abstract: The aim of this paper is to show how North Cyprus fights Covid-19 by using the basic reproduction number R_0 and effective reproduction number R_t which is the herd immunity threshold [1]. According to the Wikipedia page with title Covid-19 pandemic in Northern Cyprus, North Cyprus is the first country in Europe to free from Covid-19. One of the important reasons of this is that the government decided for tackling Covid-19 pandemic by using R_0 and R_t daily. For R_0, we constructed a new SEIR model by using the real data for North Cyprus. From March 11, 2020 to May 15, 2020 R_0, varies from 0.65 to 2.38.

Keywords: Covid-19, Northern Cyprus, epidemics, mathematical model.

2010 Mathematics Subject Classification: 46N30, 46L53, 97M10

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Mathematical modelling of HIV infection with the effect of horizontal and vertical transmissions

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Abstract: Human Immuno-deficiency Virus(HIV) intends to reduce or destroy the human defense mechanisms to prevent fighting with infections or any other diseases and the progression of this virus occurred as a result of infecting the CD4+ T-cells of the organism [1]. In this study, we developed a mathematical model to study the transmission dynamics of HIV infection and analyzed the effect of horizontal and vertical transmission in Turkey. We fit the model by using confirmed HIV cases of both vertical and horizontal transmission between 2011 and 2018. By using the next generation operator, we obtained the basic reproduction number of the model which shows whether the disease persists or dies out in time. Our analysis shows that the model is locally asymptotically stable when the basic reproduction number $R_0 < 1$ and unstable when $R_0 > 1$. Further, the most sensitive parameters, that are efficient for the control of the infection, obtained by using forward normalized sensitivity index. The results obtained with the aid of mesh and contour plots. This results show that decreasing the values of transmission rate, disease induced mortality rate, and progression rate play a significant role in controlling the spread of HIV transmission.

Keywords: HIV, AIDS, basic reproduction number, next generation matrix, sensitivity analysis..

2010 Mathematics Subject Classification: 46N30, 46L53, 97M10

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Measuring the effectiveness of biochemical autoanalyser and laser device in detection of WSLs around orthodontic brackets

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Abstract: Preventing demineralization has become a vital issue during orthodontic therapy. A clinical study found that when high-dose fluoride varnish (6% NaF and CaF) was applied around the orthodontic bracket, the mean demineralization depth was approximately 40% lower than in the control group [1].

This study used two quantitative methods to investigate the efficacy of different methods in preventing enamel demineralization around orthodontic brackets under similar in vitro conditions. The study included 90 extracted bovine incisors randomized into six groups: fluoride toothpaste(FT), non-fluoride toothpaste(NFT), fluoride varnish plus fluoride toothpaste(FV+FT), casein phosphopeptide–amorphous calcium phosphate(CPP-ACP) varnish plus fluoride toothpaste(CPP-ACP+FT), medical mineral gel plus non-fluoride toothpaste(MMG+NFT), and no intervention(control). All groups were subjected to a pH cycle. The buccal surfaces were assessed with a DIAGNOdent pen(DD) at baseline(T0) and the 19th day(T1). For each group, the mean Ca2+ loss was also measured by the colorimetric method. The current study focused on the assessment of total mineral loss using DD and colorimetric methods.

Keywords: Enamel demineralization, mineral loss, colorimetric method, DIAGNOdent pen, statistical model.

2010 Mathematics Subject Classification: 46N30, 46L53, 97M10

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A Comparative analysis of fluoride, magnesium, and calcium phosphate materials on prevention of white spot lesions around orthodontic brackets with using pH cycling model

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Abstract: One common negative side effect of orthodontic treatment with fixed appliances is the development of white spot lesions (WSLs) around brackets [1]. This study aimed to compare the efficacy of various oral hygiene practices in preventing enamel demineralization around orthodontic brackets under similar in-vitro conditions. The study included 90 extracted bovine incisors, which were randomized into 6 groups. The changes in the demineralization degree were evaluated by measuring the Ca^{+2} concentration (mg/dL) in the demineralization solution at days 5, 10, 15, 19. The procedures were carried out at the Medical Biochemistry Laboratory at Near East University Hospital. Calcium content of each beaker was measured with the Abbott Architect c8000 biochemistry autoanalyzer system using the Arsenazo III method.

Keywords: Orthodontic treatment, white spot lesions, enamel demineralization, Ca^{+2} concentration.

2010 Mathematics Subject Classification: 46N30, 46L53, 97M10

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An approximate solution of first derivatives of the mixed boundary value problem for Laplace's equation on a rectangle

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Abstract: In a rectangular domain, we discuss about an approximation of the first order derivatives for the solution of the mixed boundary value problem. The boundary values on the sides of the rectangle are supposed to have the second order derivatives satisfying the Hölder condition. Under these conditions for the approximate values of the first derivatives of the solution of mixed boundary problem on a square grid, as the solution of the constructed difference scheme a uniform error estimation of order $O(h)$ (h is the grid size) is obtained.

Numerical experiments are illustrated to support the theoretical results.

Keywords: finite difference method, approximation of the derivatives, error estimations, mixed boundary condition, Laplace equation on rectangle

2010 Mathematics Subject Classification: 65M06, 65M12, 65M22

MATHEMATICS EDUCATION

On the role of mathematics concepts proficiency in health care sector

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Abstract: In this talk, we present a survey on the role of mathematics concepts proficiency in Health sector in Nigeria. The target population comprises of students Nurses in Northern Nigeria, the students were randomly selected for the study. We formulated two research questions in the study using research self developed questionnaire. The questionnaire is based on five point Likert type scale and we used the mean analysis in analyzing the result. The Pearson's Moment Correlation and Brown's formula was used for reliability testing.

Modelling of mathematical anxiety as a three-dimensional

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Abstract: Some factors related to students maths anxiety are perceived to be important for academic failure. Such of these factors and some other factors contribute immensely to the problem of mathematics achievement that is spreading among students of Economics and Administrative Sciences. Considering such a problem as a maths anxiety, we propose a mathematical model to study how this problem is spread in Near East University at Faculty of Economics and Administrative Sciences. We have discussed about basic properties of the system. Stability analysis for both equilibria are also Given. Next, Basic reproduction number (Ro) is calculated. Our numerical findings are illustrated through computer simultaneous using MATLAB, which show the reliability of our model from the practical point of view. Students math-anxiety implications of our analytical findings are addressed critically. The model analysis reveals that all of the students (average students (N=100), below average students (N=100) and above the average students (N=100)) start to show better maths achievement on the positive way during the semester (approximately 3,5 months) .

Keywords: Maths anxiety, stability analysis, students

2010 Mathematics Subject Classification: 97D60, 97D70, 93A30

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Statistical analysis of gender gap in education and employment with examples from Nepal

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Abstract: Women are 50% of the population. If this section lags behind in education and opportunity, then half of the world is deprived of opportunities. This lag also affects science and technology which has adverse impact on society at every level. Achieve gender equality and empower all women and girls is also one of the seventeen United Nations Sustainable Development Goals. This gender gap has to be reduced to empower women and girls. So data based studies related to the gender gap in science and technology need to be conducted, as what gets measured also gets done. With 66% of people being employed in agriculture sector, the economy of Nepal is agricultural based. This workforce is mainly dominated by women. With men migrating abroad for lucrative and well paid jobs, the contribution of women in the workforce has increased from 36% in 1981 to 45% 1991. This number has further soared to 50% in 2017. But the percentage of women with ownership of land is only 20%. Here gender gap in education and employment sector in Nepal and in various science streams in two universities in particular is analyzed. University graduates are considered to be highly skilled and are employed in highly paid white collared jobs. Measurement of dynamics of change in gender gap with respect to subject of education and sector of employment is done here. The results are based on secondary data collected by the government [1] and primary data collected for this study.

Keywords: Statistical analysis, logistic regression, odds ratio, gender gap

2010 Mathematics Subject Classification: 92F05,62P25

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An alternative solution to Basel problem

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Abstract: The Basel problem was first posed in 1644 and remained open for 90 years, until Euler made his first waves in the mathematical community by solving it. During his life, he

would present three different solutions to the problem, which asks for an evaluation of the infinite series $\sum_{k=1}^{\infty} \frac{1}{k^2}$. Since then, people have continually looked for new, interesting, and enlightening approaches to this same problem. In the seminar talk given on Thursday, April 11, 2013 at the CMU Math Grad Student Seminar series 5 hand-picked solutions of the Basel problem, drawing from such diverse areas as complex analysis, calculus, probability, and Hilbert space theory. For each proof, he provide a historical context, a sketch, the proof details, and then a summary.

It is well-known that

$$(1) \qquad \sum_{k=1}^{\infty} \frac{1}{k^2} = \int_0^\infty \frac{x dx}{e^x - 1}.$$

In the present paper, a new alternative solution to Basel problem is presented. It is based on the following statment:

$$\int_0^\infty \frac{x dx}{e^x + 1} = \frac{1}{2} \int_0^\infty \frac{x dx}{e^x - 1}.$$

Keywords: Basel problem, series, sum.

2010 Mathematics Subject Classification: 11M20

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Interactive possibilities of formation of design and research activities of future applied mathematicians in the process of studying informatics

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Modernization of higher education in rapidly developing socioeconomic conditions implies a systematic reforming of future specialists' professional training. The modern society needs highly educated professionals, who are not only capable of navigating in difficult work situations, work in a team and take responsibility for the results of completed tasks, but also willing to make decisions under conditions of frequent change in technology of professional activities.

Interactive relations between the teacher and the student determine the main forms of organization of the educational process. The result is an active student activity, far from simple reproduction, endowed with a transformative, innovative, creative focus. This permits it possible to prepare a university graduate in applied sciences for systemic action in a professional situation, for working with an ever-growing flow of information received, for analyzing and designing his activities, for independent actions in conditions of uncertainty, striving for self-improvement (self-knowledge, self-control, self-esteem, self-regulation and self-development) and creative self-realization.

In this talk authors examines the formation and implementation of design and research activities of future applied mathematicians in the process of studying informatics in Turkmen State University.

Keywords: Mathematics in vocational training and career education

2010 Mathematics Subject Classification: 97M20

OTHER TOPICS

On the number of contractions in the finite full transformation semigroup

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Abstract: Let $X_n = \{1, 2, 3, ..., n\}$ and T_n be the full transformation semigroup on X_n, that is the semigroup of all full transformation of the set X_n. A transformation α in T_n is said to be a contraction if $|\alpha(x) - \alpha(y) \le |x - y|$ for all $x, y \in X_n$. The transformation α in T_n is order preserving if $x \le y \Rightarrow \alpha(x) \le \alpha(y)$ for all $x, y \in X_n$. In this paper we denote by CT_n and OCT_n the subsemigroups of T_n consisting respectively of all contraction and order preserving contraction in T_n (number of elements in each of the semigroup) and obtain formulae for the order of the two semigroups.

Keywords: Contraction, semigroup, subsemigroup, order preserving, full transformation

The revised method of ranking generalized trapezoidal fuzzy number using orthocenter of centroids

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Abstract: In this paper, a new computing approach of ranking fuzzy numbers using orthocenter of centroids to its distance from original point is proposed. The proposed method can ranks all types of fuzzy numbers including Crisp numbers with different membership functions. We apply the proposed ranking method to develop a new method to deal with fuzzy risk analysis problems as well as similarity measure both normal and non-normal generalized trapezoidal and triangular numbers. Several methods of ranking fuzzy numbers exist in the literature but the proposed method seems to be more simple and flexible than the existing approaches.

Keywords: Fuzzy numbers, centroids, orthocenter, trapezoidal, triangular, crisps numbers

On semigroups of transformations under deformed multiplication

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Abstract: Given a semigroup S and an element a of S, one always obtain a new semigroup S^a under a new operation $*$ defined by $x * y = xay$ for all $x, y \in S$ (here the product xay is the original semigroup operation of S). In this talk, we present results concerning algebraic and combinatorial properties of the semigroup S^a for the cases where S is one of finite full, partial and partial one-one transformation semigroups.

Keywords: Full, partial, partial one-one, transformation, idempotent, variant semigroup, deformed multiplication

2010 Mathematics Subject Classification: 20M20

On product of generators in finite transformation semigroups

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Abstract: The study of generating elements in finite semigroups of mappings was motivated by the work of John M. Howie in 1966 in which he proved that the singular part of the semigroup of all self mappings of a set into itself is generated its idempotent elements. This led to many interesting and important combinatorial questions in the theory of semigroups of transformations. In this talk, we present a survey of the works done in this direction concerning product of generator in finite semigroups partial, full and partial one-one transformations.

Keywords: Full, partial, partial one-one, transformation, rank, depth, gravity, defect

2010 Mathematics Subject Classification: 20M20

On the number of idempotents in certain semigroups of finite full contractions

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Abstract: Let $X_n = \{1, 2, 3, ..., n\}$ and T_n be the full transformation semigroup on X_n, that is the semigroup of all full transformation of the set X_n. A transformation α in T_n is said to be a contraction if $|\alpha(x) - \alpha(y) \leq |x - y|$ for all $x, y \in X_n$. The transformation α in T_n is order preserving if $x \leq y \Rightarrow \alpha(x) \leq \alpha(y)$ for all $x, y \in X_n$. In this talk we denote by $E(CT_n)$ and $E(OCT_n)$ the subsemigroups of T_n consisting respectively of idempotents of all contration and idempotent of all order preserving contration in T_n and obtain formula for the order.

Keywords: Contraction, full transformation, order preserving, idempotent

On the ternary semigroups of homeomorphic transformations of bounded closed sets with nonempty interior of finite-dimensional Euclidean spaces

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Abstract: This report is devoted to a characterization of bounded closed sets with nonempty interior of finite-dimensional Euclidean spaces by ternary semigroups of homeomorphic maps.

A ternary semigroup is a nonempty set T together with a ternary operation $[abc]$ satisfying the associative law $[[abc]\,de] = [a\,[bcd]\,e] = [ab\,[cde]]$ for every $a, b, c, d, e \in T$. Let R be a finite-dimensional Euclidean space with the standard topology. Let Ω_1 and Ω_2 be two bounded closed sets of R such that $Int\,(\Omega_i) \neq \emptyset$ for $i = 1, 2$. Let $B_i\,(\Omega_i)$ denote the set of all homeomorphic maps a from Ω_i to Ω_j for which there is an $n-$sized element $E_a \subset \Omega_j$ and a closed set $F_a \subset \Omega_j$ such that $a\Omega_i \subset F_a \subset IntE_a$, where $i, j = 1, 2\ (i \neq j)$. The set $B\,(\Omega_1, \Omega_2) = B_1\,(\Omega_1) \times B_2\,(\Omega_2)$ is a ternary semigroup with respect to the ternary operation

$$[(a_1, b_1)\,(a_2, b_2)\,(a_3, b_3)] = (a_1 b_2 a_3, b_1 a_2 b_3)\,.$$

Theorem 1. Let R and R' be finite-dimensional Euclidean spaces. Let Ω_1 and Ω_2 be bounded closed sets of R and let Ω_1' and Ω_2' be bounded closed sets of R' such that $Int\,(\Omega_i) \neq \emptyset$ for $i = 1, 2$. The ternary semigroups $B\,(\Omega_1, \Omega_2)$ and $B\,(\Omega_1', \Omega_2')$ are isomorphic if and only if the spaces Ω_i and Ω_i' are homeomorphic $(i = 1, 2)$.

Keywords: Euclidean space, ternary semigroup

2010 Mathematics Subject Classification: 20M20, 54C10

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Application of intuitionistic Fuzzy multisets in medical diagnosis

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Abstract: In this paper, an overview of the concept of intuitionistic Fuzzy Multisets is presented. Various operations on Fuzzy set, Intuitionistic Fuzzy set, Fuzzy Multisets, Intuitionistic Fuzzy Multiset (IFMS) were discussed. Fuzzy Multisets has wide applications in mathematics, computer science and related areas. This research focused on the real life application of IFMS. Different patients are examined, data are collected on various signs and symptoms of finite number of diseases, to ascertain the exact disease in which the patient is suffering from, Euclidean distance formla is used to analyze the data. And finally, the result from the hospital diagnosis is compared with the proposed research finding on these perspective, the result from IFMS turns out to be accurate, efficient and valid.

Keywords: Fuzzy set, Fuzzy multiset, intuitionistic Fuzzy set, intuitionistic Fuzzy multiset, Euclidean distance formula

Modeling social and economic systems

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Abstract: The emergence of economic models is based on the use of mathematics, statistics and economic approaches by putting the problems in a systematic order. There is no unique model for the different systems. According to many researchers, it is possible to solve the problems related to social, economic and traffic systems with different models. The difficulty in this area is what the model should be. Many studies have shown that there is no single model for the problems encountered in social sciences. In the literature, there are different arguments, one group advocating different models should be used at the same time to be able to compare findings and to be more confident about outcomes while the other group argues each situation has its own conditions and need to be defined by a unique and special model. The main problem with pluralistic modeling is that they can be imposed only on a few systems. Especially, as the recent financial and economic crisis shows, extremely useful solutions can be produced depending on a single, idealized and unique model. Pluralistic modeling models are generally consistent in some inconsistent situations. Empirical studies are carried out to find out whether the model is correct, best or most suitable. The simulation of economic and social structures can therefore be very difficult. For socio-economic development planning, the model can be used. Due to the nature of economic and social structures, it can be difficult to perform these analyzes and find the appropriate model. In order to overcome this long-standing theoretical puzzle, it is very important how to model it, especially for economic planning and development.

The study concludes that a paradigm shift towards a pluralistic modeling approach, which incorporates different views of the world, is overdue. Modeling can also serve many other purposes, such as understanding system responses that arise from complex system component interactions, supporting, participatory and study of the effects of complex human behavior. In this relation, it was argued that combining many different approaches can be useful, even if incoherent, in obtaining a good expression of the true life facts. In conclusion, which areas of social and economic collaboration will benefit. This study emphasize the need to be aware of the capacity, constraints and conceptual frameworks underlying the various approaches and to contribute to the integrated understanding and behavior for enhanced stability and sustainability through critical engagement with modeling for the various objectives of socio-economic systems. The study will gather data from Nigerian students in North Cyprus by use of a constructed questionnaire of about 150 respondents to collect the primary data and use an econometric application (EVIEWS 10) package program to get a better understanding of the different modelling of social and economic systems.

Keywords: Social modelling, economic modelling, design approach, models, conceptual contexts, scenario analysis.

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The awareness of users towards e-signature: a scale development study

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Abstract: The necessity of e-signature has been accepting by all organizations like finance, education, business. With the e‑ect of this, the popularity has been increasing day by day increase especially uncertainty times such as the Covid-19 pandemic. However, there is a missing gap in the literature about e-signature studies. The literature review showed that the studies on e-signature awareness and the benefits of e-signature usage in any organization or/and the person are not enough. For this reason, the purpose of the study is to develop a valid and reliable scale to use in scientific studies to understand the awareness of users towards esignature. Therefore, both validity and reliability of scale have been investigated. Through the exploratory factor analysis, the AoE-sign scale has 16 items and 2 subfactors which are "Awareness" and "Benefits". The results of factor analysis results indicated that Cronbach Alpha for Awareness was .888, for Benefits was .790 and for the overall scale was .889. Consequently, the study results demonstrated that the AoE-sign scale is reliable and valid tool and the developed scale could be used by top-level managers, in developing countries such as Cyprus, to be able to understand the awareness level of users. Moreover, they can organize staff training on e-signature to make it possible to use e-signature e‑ciently in the organizations.

Keywords: Scale development, factor analysis, e-signature, awareness, developing countries, users

Optimization of multi robots hunting game

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Abstract: This research presents multi robot cooperative hunting behavior using differential game approach. Two robots were used as pursuers while another robot is used as evader, the two robots (pursuers) try to search and surround the prey (evader) robot. The aim of the game is for the two robots to detect the evader at the minimum possible time while at the same time the evader dogged the pursuer to the maximum possible time. Differential game approach was used to construct the problem using system of ordinary differential equation. We give the required conditions for the two pursuers to catch the evader. It was also shown that the evader try to maximize the capture time, while the pursuers minimize the capture time.

Keywords: Multi robot, pursuer, evader, differential game
2010 Mathematics Subject Classification: 15A24,65F05

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Evaluation of total artificial heart using multi-criteria decision analysis

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Abstract: Heart failure is a condition that affects great parts of the world population. Heart transplantation is one of the alternatives to survive heart failure. It is not easy to find the available organ because of the lack of organ donors and the long list of the people expecting heart transplant [1]. An alternative until the organs become available is either the total artificial hearth or a ventricle assisting device. The total artificial heart is used to bridge the time until heart transplantation. There are various types of artificial hearts. In this paper, we compared various TAH devices by creating a database and in order to determine the best device in the market we used a simulation. FUZZY PROMETHEE was used as the methodology. Promising result were achieved with parametric (SynCardia) TAHs. However there are still some problems to overcome. Nevertheless, totally implantable total artificial hearts have a long way to go if they are to replace the natural heart permanently, made for destination therapy. So far, the only commercially approved and found in the market for the patient used is SynCardia TAH:

Keywords: Total artificial heart, multi-criteria decision analysis

2010 Mathematics Subject Classification: 92B20, 03B52

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Competitiveness as a criterion for innovation

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Abstract: The current stage of development of market relations in the world is characterized by a significant increase in competition, its transformation into hyper-competition. The main factors which affect the competition between manufacturers are accelerating scientific and technological progress, the attraction on a permanent basis of investments made in the development of production in order to ensure a significant increase in productivity, the introduction of innovations and the dynamic development of modern information technologies. Scientific and technological progress determines the introduction of the most effective technologies, mathematical methods of organizing and planning production, shortening the renewal cycle of products manufactured and supplied to the market. The driving force for the development of human society is the aspiration to meet growing demand through the improvement of knowledge. Competition is the competitiveness of economic subjects when their independent actions effectively restrict the ability of each of them to influence the general conditions of the goods in the market and stimulate the production of goods on the demand of the consumer. The end result of the use of the achievements of scientific and technological progress is a significant increase in the efficiency of production, the fullest satisfaction of the needs of customers in products or services, as well as a long-term and reliable presence of competitive sides in the market.

Keywords: Innovation, scientific and technological progress, competition.

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On periodic boundary value problems with an inclined derivative for a second order elliptic equation

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Abstract: In this paper, we study solvability of new classes of nonlocal boundary value problems for second-order elliptic type equation. The considered problems are multidimensional analogues (in the case of circular regions) of classical periodic boundary value problems in rectangular domains. To study the main problem, first, an auxiliary boundary value problem with inclined derivative is considered for the second order elliptic equation. The main problems are solved by reducing them to a sequential solution of the Dirichlet problem and the problem with inclined derivative. Theorems on existence and uniqueness of a solution of the problems are proved.

Note that similar problems for the Laplace and Poisson equations with normal derivatives of integer and fractional orders were studied in [1-3].

Keywords: elliptic equation, periodic problem, inclined derivative, boundary value problem, Dirichlet problem, solvability

2010 Mathematics Subject Classification: 34K06, 35J25

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Poisson's operator with integral nonlocal condition

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Abstract: For $\Pi = (0,1) \times (0,\pi)$, $f \in C(\overline{\Pi})$, $\rho \in C[0,\tau_1]$, $\tau_1 \leq 1$ we got new result on

(1)
$$\begin{cases} \Delta u(x,y) = f(x,y), \ (x,y) \in \Pi; \ u(x,0) = u(x,\pi) = 0, \ 0 \leq x < 1, \\ u(0,y) = 0, \ \ u(1,y) = \int_0^{\tau_1} \rho(x)u(x,y)dx, \ 0 \leq y \leq \pi. \end{cases}$$

<u>Theorem 1.</u> Let $\rho(x)$ satisfies one of the conditions:

(a) $\int_0^{\tau_1} \rho(x)dx < \frac{\sinh 1}{\sinh \tau_1}$ if $\rho(x)$ does not change a sign,

(b) $\int_0^{\tau_1} \rho(x)dx < \frac{\sinh 1}{\sinh \tau_0}$ if the sign changes from plus to minus in $\tau_0 \in (0,\tau_1)$,

(c) $\int_0^{\tau_1} \frac{\rho(x)+|\rho(x)|}{2} dx < \frac{\sinh 1}{\sinh \tau_1}$ if behaviour of $\rho(x)$ differs from (a) and (b). Then classical solution [1] of (1) exists and $||u||_{W_2^2(\Pi)} \leq C||f||_{L_2(\Pi)}$.

<u>Theorem 2.</u> Let $\rho \equiv 0$ in $[\tau_1, 1]$, $\int_0^{\tau_1} \rho(x)dx < (1 + \frac{4}{\pi})^{1-\tau_1-\theta}$, $\theta = \min\{\frac{\tau_1}{2}, \frac{1-\tau_1}{2}\}$, ρ satisfies (a), $i_{\tau_1} h_1 \leq \tau_1 < (i_{\tau_1}+1)h_1$ if $\tau_1 < 1$, $i_{\tau_1} + 1 = N_1$ if $\tau_1 = 1$ for

(2)
$$\begin{cases} Y_{\bar{x}x} + Y_{\bar{y}y} = f(x_i, y_j), \ (x_i, y_j) \in \Pi, \ h_1 = 1/N_1, \ h_2 = \pi/N_2, \\ Y|_{x=0} = 0, \ \ y_j \in [0,\pi], \ Y|_{y=0} = Y|_{y=\pi} = 0, \ x_i \in [0,1), \\ \sum_{i=1}^{i_{\tau_1}+1} (\rho_i Y_{i,j} + \rho_{i-1}Y_{i-1,j})h_1 - 2Y_{N_1,j} = 0, j = \overline{1, N_2-1}, \rho_i = \rho(x_i). \end{cases}$$

If $u \in C^4(\overline{\Pi})$ is the solution of (1) for $\rho \in C[0,1]$, then mesh solution Y approximates u by the second order of accuracy in terms of $h = \sqrt{h_1^2 + h_2^2}$ for $h_1 \leq c_0 h_2$ when $h_2 \to 0$ in each of the difference metrics C, W_2^2.

Keywords: integral nonlocal condition, Poisson's operator

2010 Mathematics Subject Classification: 35B45, 35J05, 65N06

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Minisymposium: Modern Problems of Mathematical Hydrodynamics

Weak solvability of one problem of fractional viscoelasticity model with memory

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Abstract: We consider the motion of a multidimensional viscoelastic continuum which subjects anti-Zener fractional constitutive law. The weak solvability of a corresponding initial-boundary value problem is established. The corresponding initial-boundary value problem has the form:

$$(1) \quad \partial v/\partial t + \sum_{i=1}^{N} v_i \partial v/\partial x_i - \mu_0 \mathrm{Div}\,\mathcal{E}(v) - \mu_1 \mathrm{Div} \int_0^t R_1(t,\tau)\,\mathcal{E}(v)(\tau, z(\tau;t,x))\,d\tau$$

$$-\mu_2 \mathrm{Div} \int_0^t R_2(t,\tau)\,\mathcal{E}(v)(\tau, z(\tau;t,x))\,d\tau + \nabla p = f, \ \mathrm{div}\,v = 0, \ (t,x) \in Q_T;$$

$$(2) \quad z(\tau;t,x) = x + \int_t^\tau v(s, z(s;t,x))\,ds, \quad 0 \le t, \tau \le T, \quad x \in \overline{\Omega};$$

$$(3) \quad v(0,x) = v^0(x), \ x \in \Omega; \ v|_{[0,T]\times\partial\Omega} = 0,$$

where kernels $R_2(t,\tau)$ are singular.

Keywords: Viscoelastic continuum, motion equations, initial-boundary value problem, weak solution, fractional derivative, regular Lagrangian flow

2010 Mathematics Subject Classification: 76A05, 35Q35

This is a joint work with V.G. Zvyagin.

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The convergence of attractors

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Abstract: The question of the attractor's existence is closely related to theorems on the existence of solutions. An effective method for studying the solvability of fluid dynamic problems and the existence of their attractors is the approximation-topological method. This method was proposed by V.G. Zvyagin and developed by him with his collaborators V.T. Dmitrienko, D.A. Vorotnikov, M.V. Turbin, A.V. Zvyagin and others. It consists of the fact that in the beginning the original initial–boundary value problem is approximated by some problem with better topological properties. Then, based on the topological degree of completely continuous, or condensing vector fields, or mappings satisfying the alpha condition and a priori estimates of the solutions, the solvability of these approximation problems is established. Approximation equations contain approximation parameters, and these equations move into the original, unperturbed equations as the approximation parameter tends to zero. Once the solvability of the approximation problems is proved, the solvability of the original problem is established using the passage to the limit.

For the solutions to the original problem usually it is possible to obtain some estimates of the dissipative type. Based on them, the trajectory space is constructed, for which the existence of trajectory and global attractors of both the approximation problem and the original problem is proved. Approximation equations have usually more natural properties of continuous dependence of solutions on the right-hand side and initial conditions, i.e. for small changes in the initial conditions and the right-hand side of the equation, a small change in the set of solutions is obtained. The presence of this property makes it possible to apply various approximate methods to find attractors of approximation problems and to study their convergence to attractors of the original system. Such an approach can be useful for the approximate calculation of the attractors for the original problem.

Keywords: Approximation-topological method, attractor's existence, trajectory and global attractors

2010 Mathematics Subject Classification: 35B41, 35Q35, 76D03

About optimal feedback control problem for motion model of nonlinearly viscous fluid

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Abstract: The motion of an incompressible nonlinearly viscous fluid in a bounded domain $\Omega \subset R^n$, $n = 2,3$, on the time interval $[0,T]$ ($T < \infty$) is described by the following initial–boundary value problem

$$(1) \qquad \frac{\partial v}{\partial t} + \sum_{i=1}^{n} v_i \frac{\partial v}{\partial x_i} - \left[2\mu\big(I_2(v)\big)\varepsilon(v) \right] + p = f,$$

$$(2) \qquad v = 0, \qquad v|_{t=0} = v_0(x), \qquad v|_{\partial\Omega \times [0,T]} = 0.$$

Here $v(x,t)$ is a vector–function of the velocity of a fluid particle at a point $x \in \Omega$ at a time $t \in [0,T]$; p is a pressure function in a fluid; f is the density of external forces; ε is the strain rate tensor
$\varepsilon(v) = \big(\varepsilon_{ij}(v)\big)$, $\varepsilon_{ij}(v) = \frac{1}{2}\big(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\big)$, tensor $I_2(v)$ is determined by the equality

$$I_2^2(v) = \varepsilon(v) : \varepsilon(v) = \sum_{i,j=1}^{n} \left[\varepsilon_{ij}(v)\right]^2.$$

In this presentation solutions existence to the feedback control problem for a nonlinearly viscous fluid model (1),(2) is studied. Also the existence of an optimal solution to the problem under consideration that gives a minimum to a given bounded quality functional is proved.

Keywords: Optimal feedback control problem, existence theorem, nonlinear viscous fluid

2010 Mathematics Subject Classification: 76D55, 49J20, 35Q35

Minisymposium: Actual mathematical problems in control theory

On the solvability of the synthesis problem at optimal control of oscillatory processes described by integro-differential equations

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Abstract: In the paper synthesis problem is investigated of optimal control in the optimization of oscillation processes when equation of boundary value problem contains the Fredholm integral operator.

The investigation is conducted by methodology of professor A.I. Egorov with developed on basis of Bellman scheme. At this using the notion of a generalized solution of boundary value problem and the notion of the Frechet differential for Bellman functional. For Bellman functional is obtained integro-differential equation in partial derivatives with complicated structure. Indicated the structure of solution. An algorithm is developed for constructing the synthesizing optimal control.

A pursuit differential game problem on a closed convex subset of l_2

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Abstract: We study a pursuit differential game problem with finite number of pursuers and one evader on a nonempty closed convex subset of l_2. Players' motions are described by ordinary differential equations and control functions of the pursuers and evader are subject to integral constraints. Pursuit is said to be completed if the geometric positions of a pursuer and the evader coincide. We formulated and proved theorems, each of which provides a condition for completion of pursuit. Consequently, strategies of the pursuers that ensure completion of pursuit are constructed. Furthermore, illustrative example was given to demonstrate the result.

Keywords: Pursuit, integral constraint, closed convex set

Optimality conditions for systems of forward backward doubly SDEs of mean-field type

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Abstract: In this work, we consider a control problem for systems driven by a non-linear forward-backward doubly stochastic differential equations of meanfield type (MF-FBDSDEs). In particular, we establish necessary as well as sufficient optimality conditions for the existence of both optimal relaxed control and optimal strict control for this kind of control problem of mean-field forward-backward doubly SDEs.

Keywords: Mean-field, forward backward doubly stochastic differential equations, strict control, relaxed control, existence, optimality conditions.

2010 Mathematics Subject Classification: 60H10, 60G55, 93E20

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On the solvability of nonlinear integral equations

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Abstract: The article explores general nonlinear equations with a parameter. Sufficient conditions for the existence of a solution of nonlinear integral equations in the form of the sum of two functions for individual values of the parameter are found.

Considered the nonlinear integral equation

$$(1) \qquad A[\varphi(x)] = \lambda \int_a^b h(x)F[t, \varphi(t)]dt + f(x),$$

where $\varphi(x)$ an unknown function; $\lambda \in (-\infty, \infty)$ parameter; $f(x), h(x)$ are given continuous functions, defined at a given interval $[a, b]$; $A[\varphi(x)]$ and $F[x, \varphi(x)]$ are given continuous functions, which have continuous derivatives with respect to a functional variable φ.

Investigated the solvability of the problem where it is required to find the parameter values λ, for which the equation (1) has a non-trivial solution.

The solution to equation (1) is sought in the form

$$(2) \qquad \varphi(x) = \varphi_0(x) + \lambda u(x),$$

where functions $\varphi_0(x), u(x)$, and a parameter λ to be determined. Suppose that function (2) is a solution of equations (1). Then the identity holds

$$(3) \qquad A[\varphi_0(x) + \lambda u(x)] = \lambda \int_a^b h(x)F[t, \varphi_0(x) + \lambda u(x)]dt + f(x).$$

Further research was carried out according to the Lagrange finite increment formula.

Keywords: nonlinear integral equation, finite increment formula, resolvent values

2010 Mathematics Subject Classification: Primary 49J20; Secondary 35K20

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On solvability of the nonlinear optimization problem with the limitations on the control

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Abstract: In the article the solvability of the problem of optimal control of oscillatory processes described by the integro-differential equation with the Fredholm operator, with given control limitation is investigated.

Consider the problem of minimizing the functional

$$(1) \qquad J[u] = \int_0^1 [V(T,x) - \xi(x)]^2 dx + \beta \int_0^T p[t, u(t)]dt, \beta > 0$$

on the set of solutions of boundary value problem

$$V_{tt} = V_{xx} + \lambda \int_0^T K(t,\tau)V(\tau, x)d\tau + g(t,x)f[u(t)], \quad 0 < x < 1, \ 0 < t \le T,$$

$$(2) \qquad V(0,x) = \psi_1, \ V_t(0,x) = \psi_2, \quad 0 < x < 1,$$

$$V_x(t,0) = 0, V_x(t,1) = 0, \ 0 < t < T.$$

Here the control $u(t)$ is an element of a Hilbert space $H(0,T)$, i.e. $u(t) \in H(0,T)$; $f[u(t)] \in H(0,T)$ is external influence function, which is nonlinear and monotonic with respect to the functional variable $u(t)$ i.e.

$$(3) \qquad f_u[u(t)] \ne 0, \ \forall t \in [0,T]$$

functions $p[t, u(t)] \in H(0,T)$, $\xi(x) \in H(0,1)$, $g(t,x) \in H(Q)$, $Q = (0,1) \times (0,T)$, $\psi_1(x) \in H_1(0,1)$, $\psi_2(x) \in H(0,1)$, $K(t,\tau) \in H(D)$, $D = (0,T) \times (0,T)$ and number

$$K_0 = \int_0^T \int_0^T K^2(t,\tau)d\tau dt$$

are considered as given, λ is a parameter, $H_1(0,1)$ is the first order Sobolev space, T is fixed time point.

In this task, the desired control $u^0(t) \in H(0,T)$ is searched among the elements of the set

$$(4) \qquad M = \{u(t) \in H(0,T) | \ f_u[u(t)] \ne 0, \ p_u[t, u(t)] = 0\}$$

which minimizes the functional (1) together with the corresponding solution $V(t,x)$ of the boundary value problem (2).

It is established that the desired control is among the solutions of the nonlinear Fredholm integral equation of the first kind. Sufficient conditions for the existence of a solution of nonlinear optimization problem are found.

Keywords: Integro-differential equation, generalized solution, functional, optimality condition, nonlinear Fredholm integral equation of the first kind.

2010 Mathematics Subject Classification: Primary 49J20; Secondary 35K20

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