Numerical solution of the reverse parabolic problem with integral and second kind boundary condition

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Abstract: The papers [1]-[3] are devoted to study well-posedness of reverse parabolic problems with nonlocal condition and their approximations.

Let Ω be unit open cube in R^n, $S = \partial\Omega$, $\overline{\Omega} = S \cup \Omega$, and $f : (0,1) \times \Omega \to R, a_r : \Omega \to R, \mu : [0,1] \to R, \psi : \overline{\Omega} \to R$, be given functions, σ is known number ($\sigma > 0$), $\forall x \in \Omega,\ \forall r = 1, ..., n, a_r(x) \geq a_0 > 0$.

In this study, we consider approximation of the following reverse parabolic problem with integral and second kind boundary condition

$$(1)\qquad \begin{cases} u_t(x,t) + \sum\limits_{r=1}^{n} \left(a_r(x) u_{x_r}(x,t)\right)_{x_r} - \sigma u(x,t) = f(x,t), \\ x = (x_1, .., x_n) \ \in \Omega, 0 < t < 1, \\ u(x,1) = \int\limits_0^1 \mu(s) u(x,s) ds + \psi(x),\ x \in \overline{\Omega}; \\ \frac{\partial u}{\partial \vec{n}}(x,t) = 0,\ x \in S, 0 \leq t \leq 1. \end{cases}$$

We study the first order difference scheme for approximate solution of reverse problem. Stability and coercive stability estimates for solution of difference scheme are obtained. Numerical results together with an explanation on the realization in one and two dimensional test examples are presented.

Keywords: Difference schemes, reverse parabolic problem, stability, integral condition

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