On approximation of first order derivatives of complex-valued functions by finite differences

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Abstract: Boundary value problems for partial differential equations involving complex valued functions have important applications in a broad sense ([1–5]). The theory of finite difference method in case of real valued function and its applications to solve boundary value problems for partial differential equations is described in [6]. Complex step method for computing derivatives of real valued functions by introducing a complex step in a strict sense is considered in [7, 8] (see also references therein).

In this presentation, we generalize the well known finite difference method to compute derivatives of real valued function to approximate of complex derivatives w_z and $w_{\overline{z}}$ for complex valued function w. Exploring different combinations of terms, we derive several approximations to compute the first order derivatives of complex valued function w. The first, second, third and fourth order of accuracy finite differences to calculate derivatives are studied. Error analyses in test examples are carried out by using Matlab program.

Keywords: finite difference, complex-valued function, approximation, error.

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