Well-posedness of Neumann-type elliptic overdetermined problem with integral condition

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Abstract: In this work, we consider the following Neumann type elliptic overdetermined problem with integral condition to find a function $u \in C^2\left([0,T],\ H\right) \cap C\left([0,T],\ D(A)\right)$ and an element $p \in H$:

$$\begin{cases} -u_{tt}(t) + Au(t) = f(t) + p, \quad 0 < t < T, \\ u_t(0) = \varphi,\ u_t(T) = \int\limits_0^T \alpha\left(\lambda\right) u_\lambda(\lambda) d\lambda + \psi,\ u(\lambda_0) = \zeta. \end{cases}$$

where A is a selfadjoint and positive definite operator in an arbitrary Hilbert space H, smooth function $f(t)$, the elements $\varphi, \zeta, \psi \in D(A)$ and number $\lambda_0 \in (0,T)$ are given. Let the given smooth scalar fuction $\alpha\left(t\right)$ be under condition $\int\limits_0^T \left|\alpha\left(\lambda\right)\right|\ d\lambda < 1$.

In the papers [1–5] well-posedness of various overdetermined elliptic type differential and difference problems are studied.

Abstract results on stability, almost coercive stability and coercive stability estimates for the solution of this problem are established. Later, the abstract results are used to establish well-posedness of overdetermined problem multi-dimensional elliptic equation with integral boundary condition.

Keywords: inverse problem, well-posedness, stability, coercive stability, overdetermination.

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