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Effects of random material and geometrical properties on structural safety of steel–concrete composite systems

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SUMMARY

A stochastic finite element-based algorithm for probabilistic analysis of structural systems made of composite sections with random material and geometrical properties under earthquake forces is proposed in this paper. Uncertainties in the structural parameters can be taken into account in this algorithm. For the perturbation-based stochastic finite element method, only the first two moments of random variables need to be known, and is numerically much more efficient and feasible than simulation techniques. The efficiency and accuracy of the proposed algorithm are validated by comparison with results of Monte Carlo simulation method. A summary of stiffness matrix formulation and perturbation-based stochastic finite element dynamic analysis formulation of structural systems made of composite sections is given. These are followed by suitable numerical examples, which indicate that employment of such a dynamic stochastic finite element method leads to significant economical, efficient and accurate solutions for the dynamic analysis of composite structures with stochastic parameters under earthquake forces.

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KEY WORDS: stochastic finite element method; perturbation-based stochastic finite element method; composite structures; stiffness matrix; Monte Carlo simulation

1. INTRODUCTION

Recent trends in the construction of moment-framed buildings show the increased use of steel, reinforced concrete and composite steel–concrete members functioning together. Such systems make use of each type of member in the most efficient manner to maximize the structural and economic benefits. In general, since composite systems realize the most efficient use of steel, reinforced concrete and composite members in a structural system, this type of construction is often more economical than the traditional either all-steel or all-reinforced concrete construction. Composites are lightweight and flexible in net shape fabrication. Therefore, the construction time, part redundancy and labor cost will be less for building composite structures due to ease of handling.

Probabilistic analysis is an appropriate tool for the analysis of structural systems with randomly varying material and/or geometric properties. The inclusion of the parameter uncertainty in the analysis of structures is essential for a more complete understanding of the structural behavior. Such uncertainty can arise because of the numerous assumptions made when modeling the geometry, material properties, magnitudes of loads and boundary conditions of the structural members. Because of these uncertainties, the deterministic method can remain insufficient in a lot of structural

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system analyses. In the literature, several techniques have been proposed to evaluate the response of structures with uncertain parameters. Monte Carlo simulation (MCS) is the most used among the stochastic analysis methods for structural problems [1]. It lies on the generation of a defined number of samples of the uncertain parameters and on the solution of the corresponding deterministic problems. However, as the number of degrees of freedom of the structure and the number of uncertain parameters increase, the Monte Carlo structural analyses become very heavy from a computational point of view, and, in some cases, the computational effort makes them inapplicable [2]. For this reason, some non-statistical alternative procedures have been proposed in the literature [2–6]. On the other hand, stochastic finite element method (SFEM), which is one of the probabilistic analysis methods, increases its reliability day-by-day. This method is applied in several fields in civil engineering, especially simple or semi-complex structural systems. Most of them are based on perturbation techniques, so that the SFEM is often identified as the classical finite element method (FEM) coupled with a perturbation approach. The perturbation-based stochastic finite element method (PSFEM) is applied in several fields in civil engineering, especially simple or semi-complex structural systems.

The PSFEM has been widely applied to stochastic problems for analyzing uncertain systems since it usually requires low computational resources. This method presents a version of the FEM, which accounts for uncertainties in both the geometry and/or material properties of the structure, as well as in the applied load. In the perturbation-based stochastic finite element analysis, only the first two moments of random variables need to be known, whereas statistical techniques such as the MCS method generally require knowledge of probability density functions (PDF) that are usually not available in practice [2].

Composite structures made of steel–concrete beams and columns are nowadays common solutions in the design of seismic resistant structures. Therefore, in the last 10 years, growing attention has been given to finite element modeling and analysis of steel–concrete composite structures [7]. Although there is an extensive literature on deterministic analysis of stiffened laminated plates, shell, fiber reinforced polymer beams and on composite concrete slabs stiffened by steel beams [8–13], the technical literature is not adequate on the stochastic dynamic analysis of structural systems made of composite sections. Starting from the knowledge of structure geometry, the boundary conditions and material properties, the mass, stiffness and damping distribution of the structure are expressed in a matrix form. To identify changes in the material and geometrical properties of a structure, stochastic finite element is often required. In particular, the effects of uncertainties in material and geometrical properties and stacking sequences on structural safety of composite structures should be studied thoroughly if highly reliable composite structures are to be designed.

Ngah and Young [14] studied an application of the spectral stochastic finite element method (SSFEM) for predicting the performance of a composite structure with variable material constitutive properties. It was observed that the SSFEM is applicable over a wider range of material variability (standard deviation up to about 24% of mean). Antonio and Hoffbauer [15] performed structural responses of statically loaded composite plate and shell structures with randomness in material properties. In addition, it has been observed that there are very limited works available regarding the PSFEM aspects of structures having composite sections. Very few researchers [4, 16, 17] studied the PSFEM with random variable material and geometrical properties of composite structures. Kaminski [4] proposed to generalize *n*th order stochastic perturbation technique that can be applied to solve some boundary value or boundary initial problems in computational engineering with random parameters. He concluded that stochastic convergence of this methodology strongly depends on the coefficient of variation (COV) of the input random variable. Kaminski and Kleiber [16] carried out the stochastic second order and second moment perturbation analysis for homogenization of the two-phase periodic composite structure. Ganesan and Kowda [17] investigated the buckling of prismatic composite beam-columns with the objective of determining the mean values, mean square values, and standard deviation values of the buckling loads. The randomness in the material and geometric properties of the laminated beam-columns is modeled using homogeneous stochastic fields in space by them. The perturbation method is employed in the context of stochastic analysis.

The focus of the present paper is to perform the effects of random material and geometrical properties on structural safety of steel–concrete composite structural systems by using the PSFEM and MCS method. For this reason, dynamic problems of beam-type structures with parameters described deterministically and/or stochastically proposed by Kleiber and Hien [2] were programmed in FORTRAN language by the authors and incorporated into a general-purpose computer program for dynamic deterministic and stochastic analysis of medium and large-scale three-dimensional frames. The program is modified for the stochastic dynamic analysis of composite frame systems based on the PSFEM and is used in the stochastic dynamic analysis of the composite system. Then, this program is combined to the MCS method. The connection parameters, material and geometrical properties are assumed to be random variables in the analyses. The Kocaeli earthquake that occurred in 1999 is considered as a ground motion. In this paper, earthquake phenomenon was adopted to be deterministic. During stochastic analysis, displacements and internal forces of the systems are obtained from PSFEM and MCS method by using different uncertainties of material (elastic module and mass density) and geometrical (cross-sectional area) characteristics. First, the stochastic analysis results acquired from all the random variables are compared with each other separately, and second the efficiency and accuracy of the proposed algorithm are validated by comparison with the results of MCS method.

2. FORMULATION

In this section, stiffness matrix formulation of composite systems and perturbation-based stochastic finite element dynamic analysis formulation of composite frame systems are given.

2.1. *Stiffness matrix formulation of 3-D composite frame*

The stiffness matrix formulation of the composite system is given according to References [9, 18]. For non-homogeneous cross-sections, the elastic modulus, E, is a function of position [i.e. $E = E(y,z)$]. Let the reference modulus be given by E_r. Consider a prismatic 3-D beam element of length L with an arbitrarily shaped composite cross section consisting of materials in contact, each of which can surround a finite number of inclusions, with modulus of elasticity E_j, shear modulus G_j and mass density ρ_j, occupying the regions $\Omega_j (j = 1, 2, \ldots, K)$ of the y, z plane (Figure 1). K is the number of materials. The materials of these regions are assumed homogeneous, isotropic and linearly elastic. These boundary curves are piecewise smooth, i.e. they may have a finite number of corners. Without loss of generality, it may be assumed that C_{yz} and M_{yz} are the principal systems of axes through the cross-section's centroid and shear center, respectively.

The nodal displacement vector in the local coordinate system, as shown in Figure 1, can be written as

$$\{q_B\}^{\mathrm{T}} = \{u_i, v_i, w_i, \theta_{xi}, \theta_{yi}, \theta_{zi}, u_j, v_j, w_j, \theta_{xj}, \theta_{yj}, \theta_{zj}\} \tag{1}$$

where u_i, u_j, v_i, v_j, w_i and w_j, are axial displacements at joints i and j, y-direction transverse displacement at joints i and j, z-direction transverse displacement at joints i and j, respectively. θ_{xi}, θ_{xj}, θ_{yi}, θ_{yj}, θ_{zi} and θ_{zj} are rotation displacement in x-direction at joints i and j, y-direction rotation displacement at joints i and j and z-direction rotation displacement at joints i and j, respectively.

The nodal load vector in the local coordinate system, as shown in Figure 1, can be written as

$$\{Q_\alpha\}^{\mathrm{T}} = \{N_i, Q_{yi}, Q_{zi}, M_{xi}, M_{yi}, M_{zi}, N_j, Q_{yj}, Q_{zj}, M_{xj}, M_{yj}, M_{zj}\} \tag{2}$$

where N_i, N_j, Q_{yi}, Q_{yj}, Q_{zi} and Q_{zi} are axial forces at joints i and j, y-direction shear force at joints i and j, z-direction shear force at joints i and j, respectively. M_{xi}, M_{xj}, M_{yi}, M_{yj}, M_{zi} and M_{zj} are torque in x-direction at joints i and j, y-direction bending moment at joints i and j and z-direction bending moment at joints i and j, respectively.

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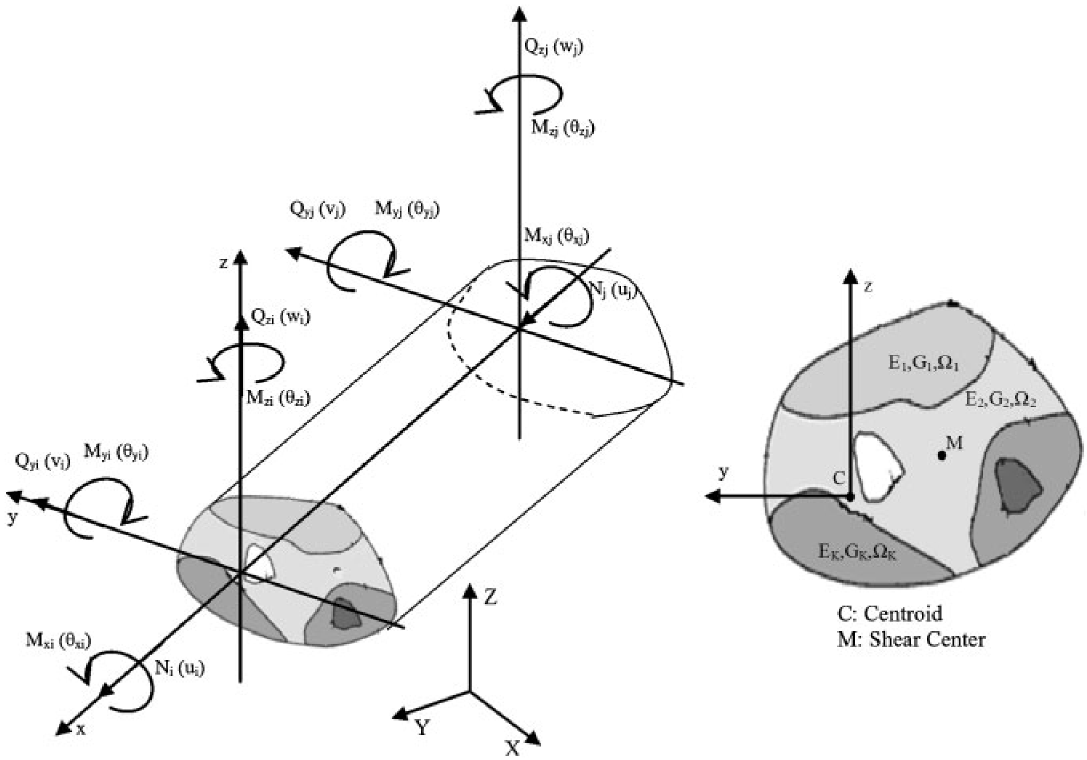

Figure 1. Prismatic beam of an arbitrarily shaped composite cross section (a) and occupying the 2-D region? (b).

The nodal displacement and load vectors given in Equations (1) and (2) are related with the 12×12 local stiffness matrix of the spatial composite beam element written as

$$[K_{\alpha\beta}]=\begin{bmatrix} k_{11} & 0 & 0 & 0 & 0 & 0 & k_{17} & 0 & 0 & 0 & 0 & 0 \\ 0 & k_{22} & 0 & 0 & 0 & k_{26} & 0 & k_{28} & 0 & 0 & 0 & k_{2,12} \\ 0 & 0 & k_{33} & 0 & k_{35} & 0 & 0 & 0 & k_{39} & 0 & k_{3,11} & 0 \\ 0 & 0 & 0 & k_{44} & 0 & 0 & 0 & 0 & 0 & k_{4,10} & 0 & 0 \\ 0 & 0 & k_{53} & 0 & k_{55} & 0 & 0 & 0 & k_{59} & 0 & k_{5,11} & 0 \\ 0 & k_{62} & 0 & 0 & 0 & k_{66} & 0 & k_{68} & 0 & 0 & 0 & k_{6,12} \\ k_{71} & 0 & 0 & 0 & 0 & 0 & k_{77} & 0 & 0 & 0 & 0 & 0 \\ 0 & k_{82} & 0 & 0 & 0 & k_{86} & 0 & k_{88} & 0 & 0 & 0 & k_{8,12} \\ 0 & 0 & k_{93} & 0 & k_{95} & 0 & 0 & 0 & k_{99} & 0 & k_{9,11} & 0 \\ 0 & 0 & 0 & k_{10,4} & 0 & 0 & 0 & 0 & 0 & k_{10,10} & 0 & 0 \\ 0 & 0 & k_{11,3} & 0 & k_{11,5} & 0 & 0 & 0 & k_{11,9} & 0 & k_{11,11} & 0 \\ 0 & k_{12,2} & 0 & 0 & 0 & k_{12,6} & 0 & k_{12,8} & 0 & 0 & 0 & k_{12,12} \end{bmatrix} \tag{3}$$

The $k_{\alpha\beta}$ coefficients of stiffness matrix of Equation (3) can be written as:

$$k_{11}=-k_{71}=k_{17}=-k_{77}=\frac{E_r A_x}{L} \tag{4}$$

$$k_{22}=-k_{28}=-k_{82}=k_{88}=\frac{12E_rG_rAI_z}{G_rAL^3+12a_yE_rI_zL} \tag{5}$$

$$k_{26}=k_{2,12}=k_{62}=-k_{68}=-k_{86}=-k_{8,12}=k_{12,2}=-k_{12,8}=\frac{6E_rG_rAI_z}{G_rAL^2+12a_yE_rI_z} \tag{6}$$

$$k_{33}=-k_{39}=-k_{93}=k_{99}=\frac{12E_rG_rAI_y}{G_rAL^3+12a_zE_rI_yL} \tag{7}$$

$$k_{59}=-k_{3,11}=-k_{53}=-k_{35}=k_{95}=k_{9,11}=-k_{11,3}=k_{11,9}=\frac{6E_rG_rAI_y}{G_rAL^2+12a_zE_rI_y} \tag{8}$$

$$k_{44}=-k_{4,10}=-k_{10,4}=k_{10,10}=\frac{G_rI_t}{L} \tag{9}$$

$$k_{55}=k_{11,11}=\frac{4E_rG_rI_yAL^2+12a_zE_r^2I_y^2}{G_rAL^3+12a_zE_rI_yL} \tag{10}$$

$$k_{5,11}=k_{11,5}=\frac{2E_rG_rI_yAL^2+12a_zE_r^2I_y^2}{G_rAL^3+12a_zE_rI_yL} \tag{11}$$

$$k_{66}=k_{12,12}=\frac{4E_rG_rI_zAL^2+12a_yE_r^2I_z^2}{G_rAL^3+12a_yE_rI_zL} \tag{12}$$

$$k_{6,12}=k_{12,6}=\frac{2E_rG_rI_zAL^2+12a_yE_r^2I_z^2}{G_rAL^3+12a_yE_rI_zL} \tag{13}$$

where E_r, G_r, A, L are the modulus of elasticity of the reference material, the shear modulus of reference material, composite cross-sectional area, composite element length, I_y, I_z are the bending moments of inertia of the composite cross-section with respect to y and z axes, respectively.

$$I_y=\sum_{j=1}^{K}\frac{E_j}{E_r}\int_{\Omega_j}z^2\,\mathrm{d}\Omega_j, \tag{14a}$$

$$I_z=\sum_{j=1}^{K}\frac{E_j}{E_r}\int_{\Omega_j}y^2\,\mathrm{d}\Omega_j \tag{14b}$$

$$A=\sum_{j=1}^{K}\frac{G_j}{G_r}\int_{\Omega_j}\mathrm{d}\Omega_j \tag{15a}$$

$$A_x=\sum_{j=1}^{K}\frac{E_j}{E_r}\int_{\Omega_j}\mathrm{d}\Omega_j \tag{15b}$$

where ignoring the torsional warping, I_t is the polar moment of inertia of the composite cross-section given as:

$$I_t=\sum_{j=1}^{K}\frac{E_j}{E_r}\int_{\Omega_j}(y^2+z^2)\,\mathrm{d}\Omega_j \tag{16}$$

It is worth noting that the reduction of Equations (14a), (14b), (15b), (16) using the modulus of elasticity E_r and of Equation (15a) using the shear modulus G_r of the reference material could be achieved using any other material, considering it as reference material.

The shear deformation coefficients; a_y, a_z are established equating the approximate formula of the shear strain energy per unit with the exact one given and are obtained as [19]:

$$a_y = \frac{A}{E_r \Delta^2} \sum_{j=1}^{K} \int_{\Omega_j} E_j ((\nabla\psi)_j - d) \cdot ((\nabla\psi)_j - d) \mathrm{d}\Omega_j \tag{17a}$$

$$a_z = \frac{A}{E_r \Delta^2} \sum_{j=1}^{K} \int_{\Omega_j} E_j ((\nabla\phi)_j - h) \cdot ((\nabla\phi)_j - h) \mathrm{d}\Omega_j \tag{17b}$$

where $(\nabla)_j \equiv i_y(\partial/\partial_y) + i_z(\partial/\partial_z)$ is a symbolic vector with i_y, i_z the unit vectors along y and z axes, respectively, Δ is given from

$$\Delta = 2(1+\upsilon) I_y I_z \tag{18}$$

υ is the Poisson ratio of the cross-section materials, d and h are vectors defined as

$$d = \left(\upsilon I_y \frac{y^2 - z^2}{2}\right) i_y + (\upsilon I_y yz) i_z \tag{19a}$$

$$h = (\upsilon I_z yz) i_y + \left(\upsilon I_z \frac{z^2 - y^2}{2}\right) i_z \tag{19b}$$

where ψ and ϕ are stress functions [9].

2.2. SFEM

In the SFEM, the deterministic finite element formulation is modified using the perturbation technique or the partial derivative method to incorporate uncertainty in the structural system. Since the basic variables are stochastic, every quantity computed during the deterministic analysis, being a function of the basic variables, is also stochastic. Therefore, the efficient way to arrive at the stochastic response may be to keep account of the stochastic variation of the quantities at every step of the deterministic analysis in terms of the stochastic variation of the basic variables.

The basic idea is conceptually simple. However, the implementation in actual analysis may not be simple since it involves the computation and assembly of large matrices of partial derivatives of the various quantities in terms of the basic variables. Furthermore, devising methodologies to transform the spatially correlated random fields into the uncorrelated random fields renders the implementation more complicated. There are two fundamental ways to solve the stochastic problem: (i) analytical approach and (ii) numerical approach. Among analytical approaches, the perturbation method is widely used because of its simplicity. Numerical methods such as MCS is generally applicable to all types of stochastic problems and is often used to verify the results obtained from analytical methods. A detailed discussion of these methods is presented below.

2.2.1. PSFEM formulation. The perturbation method is the most widely used technique for analyzing uncertain systems. This method consists of expanding all the random variables of an uncertain system around their respective mean values via Taylor series and deriving analytical expressions for the variation of desired response quantities such as natural frequencies and mode shapes of a structure due to small variation of those random variables. The basic idea behind the perturbation method is to express the stiffness and mass matrices and the responses in terms of Taylor series expansion with respect to the parameters centered at the mean values. Generally, the Taylor series is expanded only to the first order. That is why this method is often referred to as first-order perturbation method.

Since the deterministic equations are valid for the MCS analysis as well, the essential differences are observed in case of perturbation-based analysis. Let us consider a deterministic equation of motion in the form of

$$M_{\alpha\beta} \ddot{q}_\beta + C_{\alpha\beta} \dot{q}_\beta + K_{\alpha\beta} q_\beta = Q_\alpha \tag{20}$$

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where $K_{\alpha\beta}, M_{\alpha\beta}, C_{\alpha\beta}$ denote the stiffness matrix, mass matrix and damping matrix, $\ddot{q}_\beta, \dot{q}_\beta, q_\beta$ denote the acceleration, velocity, displacement, respectively. The stochastic perturbation-based approach usually consists of up to the second-order equations obtained starting from the deterministic ones.

The perturbation stochastic finite element equations describing dynamic response of the single random variable system for zeroth, first and second order [2]:

Zeroth-order equation (ϵ^0 terms, one system of N linear simultaneous ordinary differential equations for $q^0_\alpha(b^0_l;\tau)$, $\alpha=1,2,\dots,N$)

$$M^0_{\alpha\beta}(b^0_l)\ddot{q}^0_\beta(b^0_l;\tau)+C^0_{\alpha\beta}(b^0_l)\dot{q}^0_\beta(b^0_l;\tau)+K^0_{\alpha\beta}(b^0_l)q^0_\beta(b^0_l;\tau)=Q^0_\alpha(b^0_l;\tau) \tag{21}$$

First-order equations, rewritten separately for all random variables of the problem (ϵ^1 terms, $\bar{N}$ systems of N linear simultaneous ordinary differential equations for $q^{;\rho}_\alpha(b^0_l;\tau)$, $\rho=1,2,\dots,\bar{N}$, $\alpha=1,2,\dots,N$)

$$\begin{aligned}&M^0_{\alpha\beta}(b^0_l)\ddot{q}^{;\rho}_\beta(b^0_l;\tau)+C^0_{\alpha\beta}(b^0_l)\dot{q}^{;\rho}_\beta(b^0_l;\tau)+K^0_{\alpha\beta}(b^0_l)q^{;\rho}_\beta(b^0_l;\tau)=Q^{;\rho}_\alpha(b^0_l;\tau)\\&-[M^{;\rho}_{\alpha\beta}(b^0_l)\ddot{q}^0_\beta(b^0_l;\tau)+C^{;\rho}_{\alpha\beta}(b^0_l)\dot{q}^0_\beta(b^0_l;\tau)+K^{;\rho}_{\alpha\beta}(b^0_l)q^0_\beta(b^0_l;\tau)]\end{aligned} \tag{22}$$

Second-order (ϵ^2 terms, one system of N linear simultaneous ordinary differential equations for $q^2_\alpha(b^0_l;\tau)$, $\alpha=1,2,\dots,N$)

$$\begin{aligned}&M^0_{\alpha\beta}(b^0_l)q^{;\rho}_\beta(b^0_l;\tau)+C^0_{\alpha\beta}(b^0_l)q^{;\rho}_\beta(b^0_l;\tau)+K^0_{\alpha\beta}(b^0_l)q^{;\rho}_\beta(b^0_l;\tau)=\{Q^{;\rho\sigma}_\alpha(b^0_l;\tau)\\&-2[M^{;\rho}_{\alpha\beta}(b^0_l)q^{;\sigma}_\beta(b^0_l;\tau)+C^{;\rho}_{\alpha\beta}(b^0_l)q^{;\sigma}_\beta(b^0_l;\tau)+K^{;\rho}_{\alpha\beta}(b^0_l)q^{;\sigma}_\beta(b^0_l;\tau)]\\&-[M^{;\rho\sigma}_{\alpha\beta}(b^0_l)q^0_\beta(b^0_l;\tau)+C^{;\rho\sigma}_{\alpha\beta}(b^0_l)q^0_\beta(b^0_l;\tau)+K^{;\rho\sigma}_{\alpha\beta}(b^0_l)q^0_\beta(b^0_l;\tau)]\}\mathrm{Cov}(b_r,b_s)\end{aligned} \tag{23}$$

$$(\cdot)^{(0)}=(\cdot)|_{\{\beta\}=\{\beta\}^{(0)}},\quad (\cdot)^{(,\rho)}_i=\frac{\partial}{\partial\beta_i}(\cdot)|_{\{\beta\}=\{\beta\}^{(0)}},\quad (\cdot)^{(,\rho\sigma)}_i=\frac{\partial^2}{\partial\beta_i^2}(\cdot)|_{\{\beta\}=\{\beta\}^{(0)}}. \tag{24}$$

where b^0_ℓ is the vector of nodal random variables, q_α is the vector of nodal displacement-type variables, τ is forward time variable, $\bar{N}$ is the number of nodal random variables. $M^0_{\alpha\beta}$, $C^0_{\alpha\beta}$ and $K^0_{\alpha\beta}$ are system mass matrix, damping matrix and system stiffness matrix, respectively. Q^0_α, q^0_β and $\mathrm{Cov}(b_r,b_s)$ are load vector, displacement and the covariance matrix of the nodal random variable, respectively. N is the number of degrees of freedom in the system. $(.)^0$ is zeroth-order quantities, taken at means of random variables, $(.)^{,\rho}$ is first partial derivatives with respect to nodal random variables, $(.)^{,\rho\sigma}$ is second partial derivatives with respect to nodal random variables.

The first two statistical moments for the random fields $b_r(x_k)$, $r=1,2,\dots,R$, are defined as

$$E[b_r]=b^0_r=\int_{-\infty}^{+\infty} b_r p_1(b_r)\mathrm{d}b_r \tag{25}$$

$$\mathrm{Cov}(b_r,b_s)=S^{rs}_b=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}(b_r-b^0_r)(b_s-b^0_s)p_2(b_r,b_s)\mathrm{d}b_r\mathrm{d}b_s \tag{26}$$

$$r,s=1,2,\dots,R$$

The latter definition can be replaced by

$$S^{rs}_b=\alpha_{b_r}\alpha_{b_s}b^0_r b^0_s\mu_{b_r b_s} \tag{27}$$

with

$$\alpha_{b_r}=\left[\frac{\mathrm{Var}(b_r)}{b^0_r}\right]^{1/2}\quad \mu_{b_r b_s}=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} b_r b_s p_2(b_r,b_s)\,\mathrm{d}b_r\,\mathrm{d}b_s \tag{28}$$

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where, $E[b_r]$, $\mathrm{Cov}(b_r, b_s)$, $\mathrm{Var}(b_r)$, $\mu_{b_r b_s}$, α_{b_r}, $p_1(b_r)$ and $p_2(b_r, b_s)$ denote the spatial expectation, covariance, variance, correlation functions, the coefficients of variation, PDF and the joint PDF, respectively. R is the random fields, which can represent randomness in the cross-sectional area, length of truss and beam members, thickness of plate and shell elements, elastic modulus and mass density of the material, etc.

The main idea behind the second-order perturbation approach to the stochastic version of the potential energy principle involves expanding all the random field variables in the problem, i.e. elastic module $C_{ijkl}[b(x_k); x_k]$, mass density $\gamma\,[b(x_k); x_k]$ and displacements $u_i[b(x_k); x_k]$ about the spatial expectations of the random fields variables $b(x_k) = \{b_r(x_k)\}$, denoted by $b^0(x_k) = \{b_r^0(x_k)\}$, via Taylor series with a given small parameter € and retaining terms up to second order. The expansions are explicitly written as;

$$C_{ijkl}[b(x_k); x_k] = C^0_{ijkl}[b^0(x_k); x_k] + \varepsilon C^{;r}_{ijkl}[b^0(x_k); x_k]\Delta b_r(x_k) + \frac{1}{2}\varepsilon^2 C^{;rs}_{ijkl}[b^0(x_k); x_k]\Delta b_r(x_k)\Delta b_s(x_k) \tag{29}$$

$$\gamma[b(x_k); x_k] = \gamma^0[b^0(x_k); x_k] + \varepsilon\gamma^{;r}[b^0(x_k); x_k]\Delta b_r(x_k) + \frac{1}{2}\varepsilon^2\gamma^{;rs}[b^0(x_k); x_k]\Delta b_r(x_k)\Delta b_s(x_k) \tag{30}$$

where

$$\varepsilon\Delta b_r(x_k) = \varepsilon[b_r(x_k) - b_r^0(x_k)] \tag{31}$$

is the first-order variation of $b_r\ (x_k)$ about $b_r^0\ (x_k)$, and

$$\varepsilon^2\Delta b_r(x_k)\Delta b_s(x_k) = \varepsilon^2[b_r(x_k) - b_r^0(x_k)][b_s(x_k) - b_s^0(x_k)] \tag{32}$$

denotes the second-order variation of $b_r(x_k)$ and $b_s(x_k)$ about $b_r^0(x_k)$ and $b_s^0(x_k)$, respectively.

All the equations, solved consequently for zeroth, first and second-order displacements, velocities and accelerations, make it possible to compute the first two probabilistic moments of the output in the form of expected values and cross-covariances of the structural response.

2.2.2. Monte Carlo method. The Monte Carlo method is quite a versatile mathematical tool capable of handling situations where all other methods fail to succeed; in structural dynamics, it has attracted intense attention only recently following the widespread availability of inexpensive computational systems. This computational availability has triggered an interest in developing sophisticated and efficient simulation algorithms. Shinozuka [19] had a pioneering role in introducing the method to the field of structural dynamics. Shinozuka used the Monte Carlo Simulation for simulating a random process as the superposition of a large number of sinusoids having a uniformly distributed random phase angle. Zhang and Ellingwood [20] used this method to obtain the effects of random material properties. However, in most studies, the Monte Carlo Simulation was used to verify the results obtained from approximate methods [21].

The Monte Carlo Simulation generates a set of random values of X according to its probability distribution function. The set can be written as $X = \{x_1, x_2, \ldots, x_N\}$ where N is the number of simulations. For each values of X,the stiffness and mass matrices are computed. At the end of N simulations, we have a random set of displacement and stress values $\{\{q_\beta\}_1, \{q_\beta\}_2, \{q_\beta\}_3, \ldots, \{q_\beta\}_N\}$, $\{\{\sigma\}_1, \{\sigma\}_2, \{\sigma\}_3, \ldots, \{\sigma\}_N\}$ for X^i. From this finite set of solutions, the expected values of displacement and stress are computed using the following formulas:

$$\mu_{\{q_\beta\}} = \frac{1}{N}\sum_{i=1}^{N}\{q_\beta\}_i \tag{33}$$

$$\mu_{\{\sigma\}} = \frac{1}{N}\sum_{i=1}^{N}\{\sigma\}_i \tag{34}$$

3. NUMERICAL EXAMPLES

The aim of this paper is to compare the stochastic dynamic responses of structural systems made of composite cross-section for random material and geometrical properties. Therefore, perturbation techniques are associated with the SFEM and MCS method, offering the practicing engineers an overview of the techniques usually employed in the analysis of the uncertain parameters of a structural system. During stochastic analysis, displacements and internal forces of the systems are obtained from PSFEM and MCS method by using different uncertainties of material and geometrical characteristics. The analysis results obtained from these two methods and all the random variables are compared with each other.

COV was selected as α=0.1 for these random variables [2]. The PSFEM is very efficient for low material variability. How it was supposed before Reference [2, 4] to be higher than 0.1–0.15 coefficients of variation (COV) of input random variables demand higher order Taylor expansion in equilibrium equations and higher order expansion of the solution.

Example 1

A frame system of 4 spans and 10 stories consisting of beams and columns that have composite cross-section subjected to earthquake ground motion (Figure 2) is selected as application (Figure 3) for the generalized PSFEM. The system also has braced members. The span length of the composite frame system is $l=4.0$ m and story height is $h=3.0$ m. The composite columns (Figure 3(b)) and composite beams (Figure 3(c)) consist of a concrete part (E_c=3.0×10^7 kPa, $G_c=1.25\times10^7$ kPa, $\rho_c=2500$ kg/m^3, $\upsilon=0.20$) (reference material) stiffened by a steel one ($E_s=2.1\times10^8$ kPa, $G_s=8.75\times10^7$ kPa, $\rho_s=7850$ kg/m^3). The column has a box-shaped closed composite cross-section as shown in Figure 3(b). The cross-section properties are computed as $A_E=A_G=0.2954$ m^2, $I_y=0.00801$ m^4, $I_z=0.00243$ m4, $I_t=0.084$ m^4. The composite beams are formed as a box-shaped composite cross-section, with uniform Poisson's ratio $\upsilon=0.20$ and damping ratios $\xi=0.05$. The cross-section properties are computed as $A_E=A_G=0.147$ m^2, $I_y=0.0014$ m^4, $I_z=0.0064$ m^4, $I_t=0.063$ m^4 (Figure 3(c)). The braced members were chosen as Euronorm IPE140 profiles. The braced members are designed as bar elements. The shear deformation coefficient for two sections is selected as $a_y=a_z=0$.

For the dynamic analysis, YPT330 component of Yarimca station records of the 1999 Kocaeli Earthquake (Figure 2) is utilized as ground motion [22]. This ground motion continued upto 35.0 s is applied to the system in a horizontal direction. The dynamic responses of the composite frame system are obtained for a time interval of 0.005 s.

The composite frame system is modeled by 310 stochastic finite elements of equal length (310 random variables, $\rho, \sigma=1, 2, \ldots, 310$; x_ρ are ordinates of the element midpoints). MCS method was simulated for 10 000 simulations.

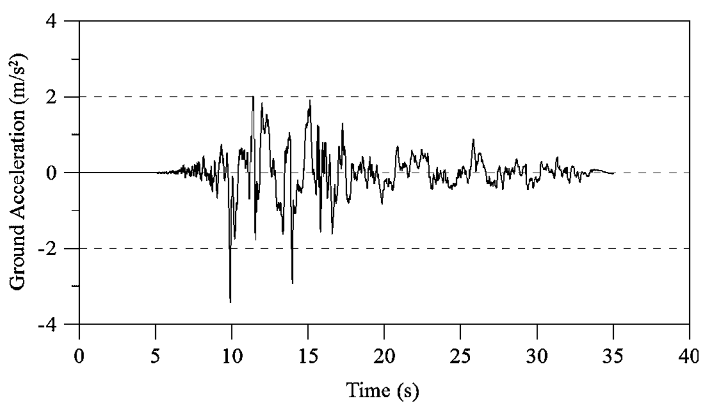

Figure 2. Acceleration time history of Kocadi earthquake (YPT330) 1999 [22].

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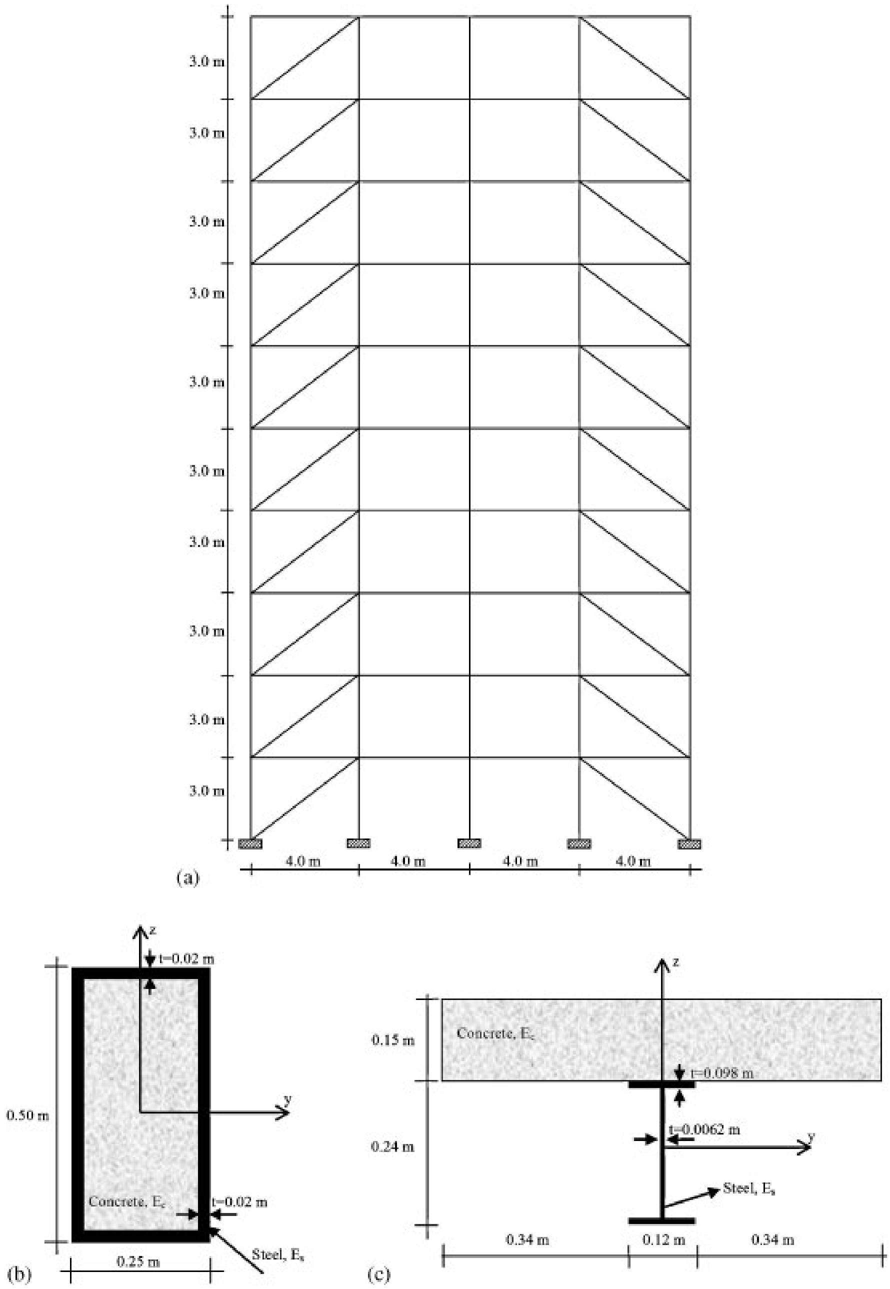

Figure 3. (a) The dimensions of the composite frame systems in Example 1; (b) the dimensions of column cross-section; and (c) the dimensions of beam cross-section.

Absolute values of maximum displacements and internal forces are determined according to PSFEM and MCS methods for the composite frame system. The analysis results obtained from Case A, Case B and Case C are compared with each other.

Case A. Elastic modulus from material properties is chosen as random variable for composite frame system. The other variables are considered as deterministic for steel–concrete composite braced frame system. The elastic modulus of composite elements is chosen as reference material's (concrete) elastic modulus. On the other hand, braced members are made of steel. The perturbation method applied uses only one variable for covariance matrix. On the other hand, MCS method considers each elastic modulus separately. For this example, during PSFEM analysis, smaller elastic modulus is selected as a random variable. This random variable is assumed to follow a normal distribution with the COV 0.1. The respective expectation and correlation function and coefficient of variation [2] for the elastic modulus E_ρ are assumed as follows:

$$E[E_\rho] = 3.0 \times 10^7 \quad \lambda = 10$$

$$\mu(E_\rho, E_\sigma) = \exp\left(-\frac{|x_\rho - x_\sigma|}{\lambda l}\right) \quad \rho, \sigma = 1, 2, \ldots, 310$$

$$\alpha = 0.10$$

where x_ρ, l and λ are ordinates of the element midpoints (n random variable, ρ, $\sigma = 1, 2, \ldots, n$), structural member length and decay factor.

Case B. Mass density from material properties is chosen as random variable for a composite frame system. The smallest of mass densities of all elements is selected as random variable. This random variable is assumed to follow a normal distribution with the COV 0.1. The respective expectation and correlation function [2] for the mass density γ_1 are assumed as follows:

$$E[\gamma_1] = 10727 \quad \lambda = 10$$

$$\mu(\rho_\rho, \rho_\sigma) = \exp\left(-\frac{|x_\rho - x_\sigma|}{\lambda l}\right) \quad \rho, \sigma = 1, 2, \ldots, 310$$

$$\alpha = 0.10$$

where x_ρ, l and λ are ordinates of the element midpoints (n random variable, $\rho, \sigma = 1, 2, \ldots, n$), structural member length and decay factor.

Case C. Cross-sectional area from geometrical properties is chosen as random variable for a composite frame system. The cross-sectional areas are different for columns, beams and braced members. The smallest of cross-section areas of all elements is selected as a random variable. The respective expectation and correlation function and coefficient of variation [2] for the cross-sectional areas (A_ρ) are assumed as follows:

$$E[A_\rho] = 0.0016 \lambda = 10$$

$$\mu(A_\rho, A_\sigma) = \exp\left(-\frac{|x_\rho - x_\sigma|}{\lambda}\right) \quad \rho, \sigma = 1, 2, \ldots, 310$$

$$\alpha = 0.10$$

where x_ρ, l and λ are ordinates of the element midpoints (n random variable, ρ, $\sigma = 1, 2, \ldots, n$), structural member length and decay factor.

Horizontal displacements along the left border of composite braced frame system according to PSFEM and MCS methods are presented in Figure 4 for Cases A-C. The total horizontal displacements values according to PSFEM of a composite frame system subjected to ground motion are smaller than those of the MCS method for all random variables (Table I). However, the displacement values obtained from the perturbation method are generally close to those calculated using MCS method, as shown in Figure 4 and Table I. At the composite braced frame system columns where maximum total horizontal displacement takes place, it can be observed that the average differences between PSFEM and MCS method are 3.86, 5.57 and 5.74%, respectively for Cases A–C. Also in Table I, the maximum horizontal displacement values that are obtained from the random cross-sectional area are larger than those calculated by the random mass density and elastic module.

The maximum bending moments and shear forces in the top joint of columns in every floor for the composite frame system are plotted in Figure 5 for Cases A–C. The maximum moments and shear forces occur in the columns at third axes for this frame system (Figure 3). It is seen from Figure 5 and Table II that values obtained from MCS and perturbation methods are much

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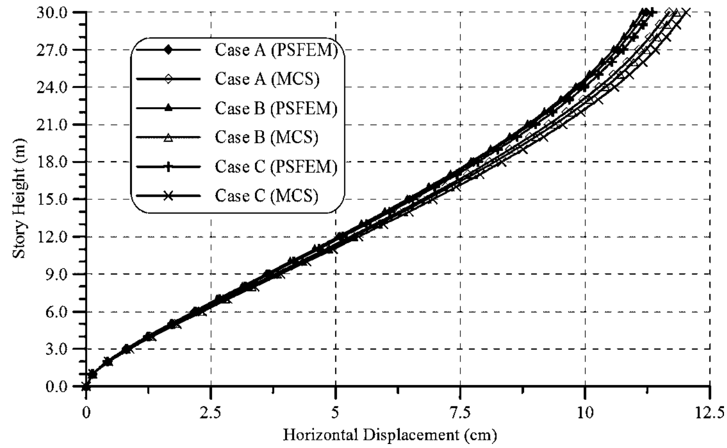

Figure 4. Horizontal displacement of the composite frame system along story height for random material and geometrical properties.

Table I. The results for the horizontal displacements along story height of composite frame system for random material and geometrical properties.

	Case A		Case B		Case C	
	PSFEM	MCS	PSFEM	MCS	PSFEM	MCS
Maximum displacement (cm)	11.226	11.682	11.348	12.023	11.949	11.829
Average displacement (cm)	6.283	6.536	6.351	6.736	6.740	6.620
Difference (According to MCS)	3.91%		5.61%		5.75%	
Average difference (According to MCS)	3.86%		5.57%		5.74%	
PSFEM average difference (According to Case A)	0.7%		1.1%			
MCS average difference (According to Case A)			3.01%		1.3%	

closer to each other. As shown in Table II, the average differences between the moments of these two methods are 1.97, 1.60 and 2.38%, respectively for Cases A–C. Likely to other results, average differences between shear forces for PSFEM and MCS method are 1.81, 1.74 and 2.05%, respectively (Table II). The mean of maxima values of internal forces obtained using MCS method are generally higher than the maximum values obtained using PSFEM. Also in this table, the maxima internal forces values that are obtained from the random elastic module are smaller than those calculated by the random mass density and cross-sectional area.

It can be seen from these tables and figures that, the mean values of dynamic responses from the three random variables are very similar to the result from the MCS method. For accurate dynamic responses, it is necessary that the analysis technique incorporate the effect of structural parameter randomness. This is of special importance for accurate stochastic dynamic analysis of composite systems, which exhibit wide dispersion in structural parameters.

Example 2

A two-dimensional arch-type bridge subjected to earthquake ground motion (Figure 2) is chosen as a second application (Figure 6) for stochastic dynamic analysis of the composite systems. The bridge consists of the steel tubes, vertical load carrying systems, piers and the deck system. The arch span consists of one-curved steel tubes. The bridge's decks are consisting of composite cross-section made from concrete and steel. The bridge's length $l = 35\,\text{m}$ and its height is $h = 13\,\text{m}$. The composite decks (Figure 6(b)) are consisting of a concrete part ($E_c = 3.0\times10^7\,\text{kPa}, G_c = 1.25\times10^7\,\text{kPa}, \rho_c = 2500\,\text{kg/m}^3, \upsilon = 0.20$) stiffened by a steel one ($E_s = 2.1\times10^8\,\text{kPa}, G_s = 8.75\times10^7\,\text{kPa}, \rho_s = 7850\,\text{kg/m}^3$) (reference material is steel). The composite decks are formed as a box-shaped composite cross-section, with uniform Poisson's ratio $\upsilon = 0.30$ and damping ratios $\xi = 0.05$. The cross-section properties are computed as $A_E = A_G = 0.0424\,\text{m}^2$, $I_y = 0.00228\,\text{m}^4$,

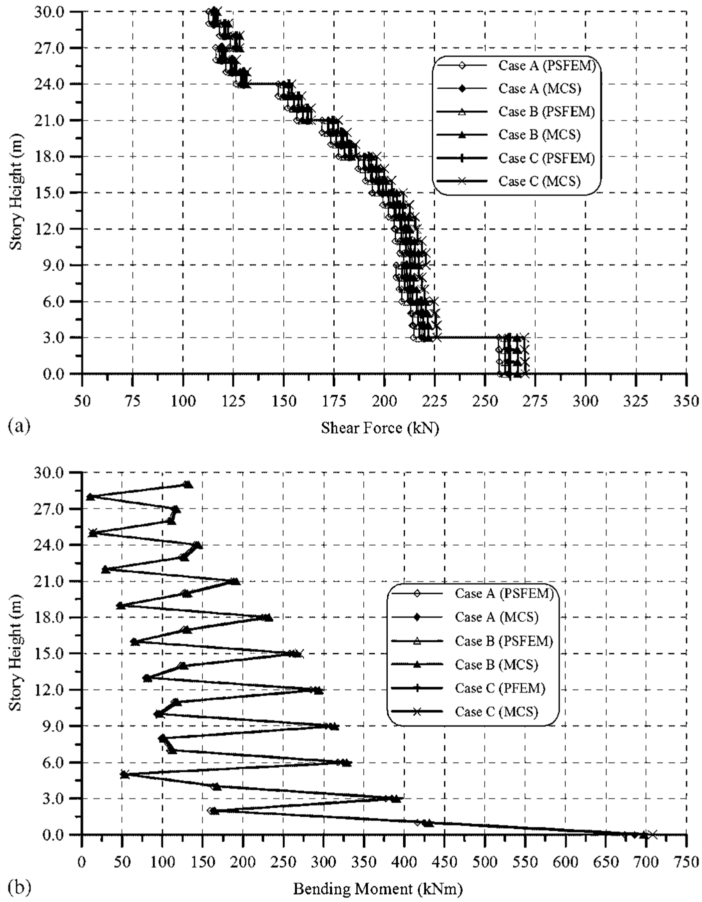

Figure 5. Maximum bending moments (a) and shear forces (b) at the top joint of the columns along story height for the composite frame system for random material and geometrical properties.

$I_z = 0.00063\,\text{m}^4$, $I_t = 0.00662\,\text{m}^4$ (Figure 6(b)). The deformation coefficients for composite deck section is selected as $a_y = a_z = 0$.

For the dynamic analysis, YPT330 component of Yarımca station records of 1999 Kocaeli Earthquake (Figure 2) is utilized as ground motion [22]. The earthquake motion continued upto 35.0 s is applied to the bridge in a vertical direction. The dynamic responses of the composite bridge are obtained for a time interval of 0.005 s.

The composite bridge is modeled by 85 stochastic finite elements (85 random variables, $\rho, \sigma = 1, 2, \ldots, 85; x_\rho$ are ordinates of the element midpoints). MCS method was simulated for 10 000 simulation.

Absolute values of maximum vertical displacements and internal forces are determined according to PSFEM and MCS method for this composite bridge system, and the analysis results compared with each other for the all random variables.

Case A. Elastic module from material properties is chosen as a random variable for composite bridge. The other variables are considered as deterministic for structural system. The elastic modulus of composite elements is chosen as reference material (steel). The other sections are made of only steel. For this reason, it is assumed that random variable is steel's elastic modulus.

$$E[E_\rho] = 2.1 \times 10^8 \quad \lambda = 10$$

$$\mu(E_\rho, E_\sigma) = \exp\left(-\frac{|x_\rho - x_\sigma|}{\lambda l}\right) \quad \rho, \sigma = 1, 2, \ldots, 85;$$

Table II. The results for shear forces and bending moments of composite frame system for random material and geometrical properties.

		Case A		Case B		Case C	
		PSFEM	MCS	PSFEM	MCS	PSFEM	MCS
Shear forces	Maximum shear force (kN)	257.113	261.516	260.073	266.121	262.393	269.875
	Average shear force (kN)	184.315	187.723	187.275	190.766	189.140	193.373
	Difference (According to MCS)	1.68%		2.27%		2.77%	
	Average difference (According to MCS)	1.81%		1.74%		2.06%	
	PSFEM average difference (According to Case A)			1.6%		2.62%	
	MCS average difference (According to Case A)			1.62%		3.01%	
Bending moments	Maximum bending moment (kNm)	673.614	685.880	676.564	697.216	677.014	707.905
	Average bending moment (kNm)	168.647	171.848	171.094	174.552	171.555	177.026
	Difference (According to MCS)	1.79%		2.96%		4.36%	
	Average difference (According to MCS)	1.97%		1.60%		2.38%	
	PSFEM average difference (According to Case A)			1.45%		1.72%	
	MCS average difference (According to Case A)			1.60%		3.01%	

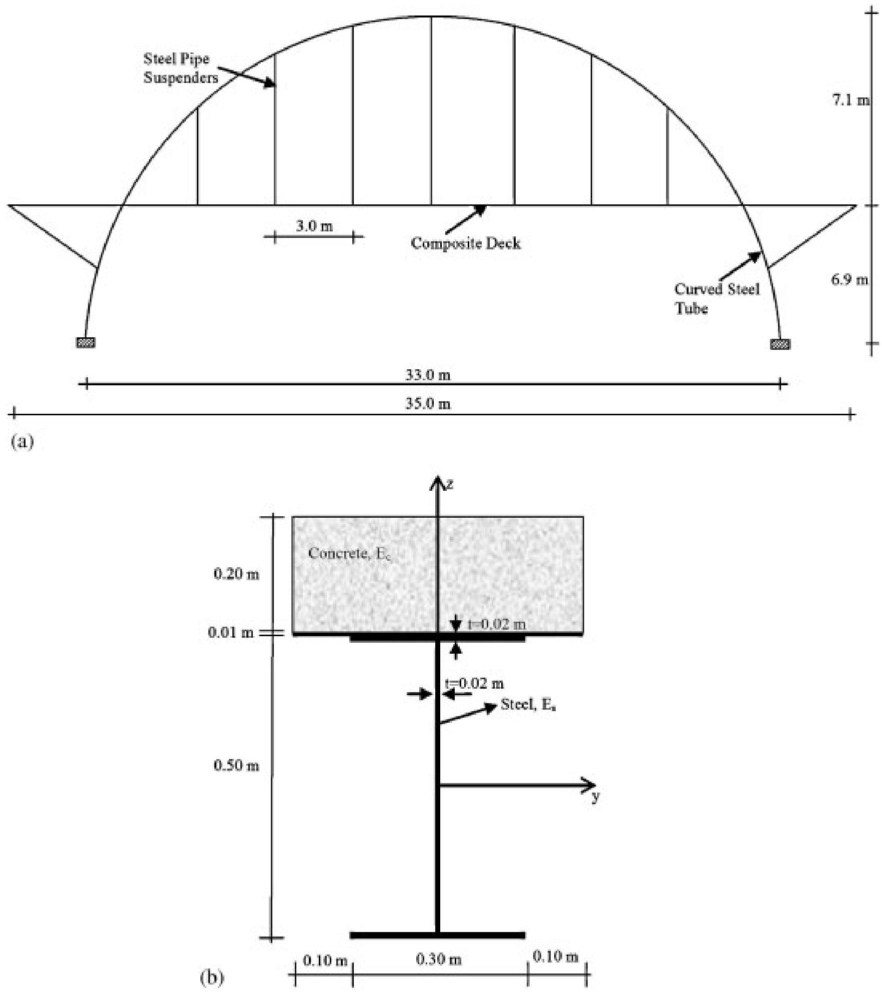

Figure 6. (a) The dimensions of the bridge in Example 2 and (b) the dimensions of deck of the bridge.

where x_ρ, l and λ are ordinates of the element midpoints (n random variable, $\rho, \sigma = 1, 2, \ldots, n$), structural member length and decay factor.

Case B. Mass density from material properties is chosen as random variable for composite bridge system. The other variables are considered to be deterministic for this system. The mass densities are different for decks and arch. The perturbation method applied uses only one variable for covariance matrix. However, the smallest of mass densities of all elements, 7850 kg/m^3, is selected as variable. The respective expectation and correlation function for the mass density (γ_p) are assumed as follows:

$$E[\gamma_1] = 7850 \quad \lambda = 10$$

$$\mu(\rho_\rho, \rho_\sigma) = \exp\left(-\frac{|x_\rho - x_\sigma|}{\lambda l}\right) \quad \rho, \sigma = 1, 2, \ldots, 85$$

where x_ρ, l and λ are ordinates of the element midpoints (n random variable, $\rho, \sigma = 1, 2, \ldots, n$), structural member length and decay factor.

Case C. Cross-section area from geometrical properties is chosen as random variable for composite frame system. The other variables are considered to be deterministic for this system. The cross-section areas are different for suspenders, decks and arch members. The perturbation method applied uses only one variable for covariance matrix. The smallest of cross-sectional areas

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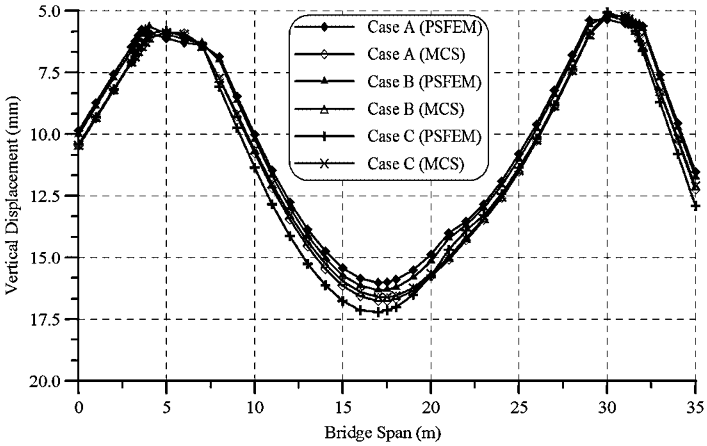

Figure 7. Vertical displacements along the bridge deck for random variable material and geometrical properties.

Table III. The results for the vertical displacements along bridge deck for random material and geometrical properties.

	Case A		Case B		Case C	
	PSFEM	MCS	PSFEM	MCS	PSFEM	MCS
	PSFEM	MCS	PSFEM	MCS	PSFEM	MCS
Maximum displacement (mm)	16.012	16.755	16.314	16.600	17.215	16.620
Average displacement (mm)	9.537	10.083	9.662	10.024	10.224	10.028
Difference (According to MCS)	4.45%		1.71%		3.69%	
Average difference (According to MCS)	5.16%		3.91%		2.59%	
PSFEM average difference (According to Case A)			1.31%		7.20%	
MCS average difference (According to Case B)	0.59%				0.04%	

of all elements is selected as variable. The respective expectation and correlation function and COV for the cross-sectional areas (A_ρ) are assumed as follows:

$$E[A_\rho] = 0.00018 \quad \lambda = 10$$

$$\mu(A_\rho, A_\sigma) = \exp\left(-\frac{|x_\rho - x_\sigma|}{\lambda}\right) \quad \rho, \sigma = 1, 2, \ldots, 85$$

where x_ρ, l and λ are ordinates of the element midpoints (n random variable, $\rho, \sigma = 1, 2, \ldots, n$), structural member length and decay factor.

Vertical displacements along the bridge deck according to PSFEM and MCS methods are presented in Figure 7 for Cases A–C. The displacement values obtained from the PSFEM are generally close to those calculated using MCS method, as shown in Figure 7 and Table III. At the bridge deck where maximum total vertical displacement takes place, it can be observed that the average differences between PSFEM and MCS method are 5.16, 3.91 and 2.59%, respectively for Cases A–C (Table III). The total vertical displacements of the deck of composite bridge are close to each other for all random variables. However, means of maximum vertical displacement of the deck of bridge obtained for Case C random cross-sectional area are larger than those of Cases A and B random variables, as shown in Table III.

Maximum moments and shear forces of deck spans along the bridge system are plotted in Figure 8 for Cases A–C. It is seen from Figure 8 and Table IV that values obtained from PSFEM and MCS methods are closed to each other. As shown in Table IV, the average differences between the moments of these two methods are 1.36, 1.52 and 1.52%, respectively for Cases A–C. Likely

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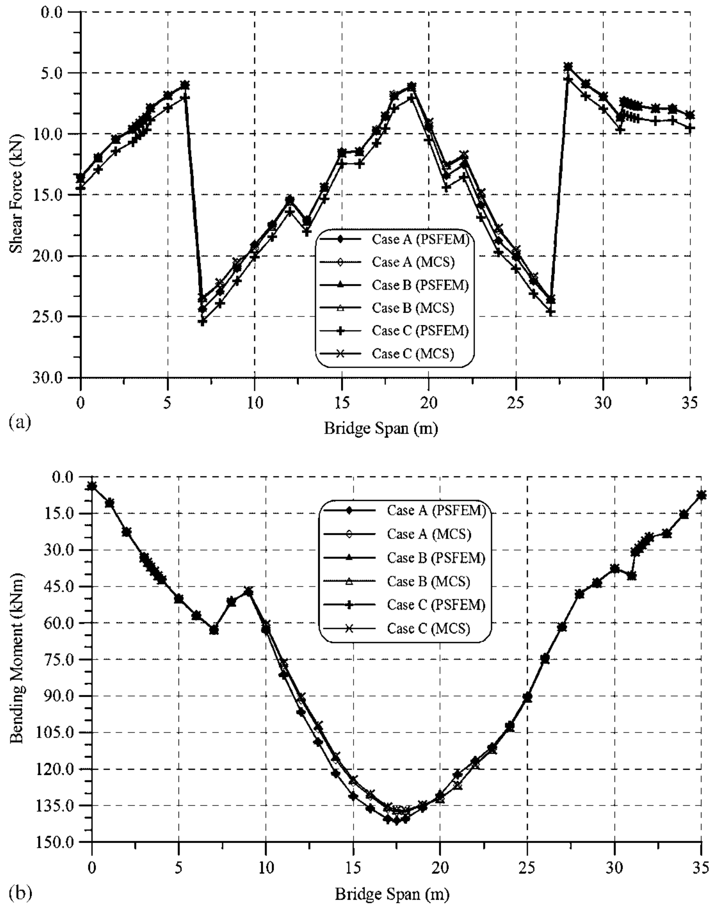

Figure 8. Maximum absolute bending moments (a) and shear forces (b) in the first joint of each element of bridge's deck for random variable material and geometrical properties.

to other results, average differences between shear forces for PSFEM and MCS method are 1.42, 1.65 and 5.05%, respectively (Table IV).

These figures show that displacements and internal forces, which occur on the deck of the bridge, are very close to all random variables (elastic module, mass density and cross-section area) for PSFEM. Similar variation is also observed for MCS method of deck of bridge (Table IV). Therefore, it can be said that PSFEM could be used instead of MCS method.

There is a significant difference in CPU time between the proposed method and the MCS method, while the results obtained from the proposed method are very close to those obtained from the MCS method. Referring to CPU time, PSFEM requires a much smaller amount of time (8 s for bridge system (Figure 6) and 8 min for frame system (Figure 3)) than does the direct MCS (5 h for bridge system (Figure 6) and 36 h for frame system (Figure 3)) is close to time other methods do. It is seen that as the number of degrees of freedom of the structure and the number of uncertain parameters increase, the structural analyses based on MCS 15 method become very heavy from a computational point of view. Therefore, PSFEM is indeed practical and efficient.

The MCS method is suited for the incorporation within the existing finite element code. However, it was found that computational time is a crucial issue for this method to deal with practical dynamic problems. The PSFEM is presented in the simplest form for implementation into an existing finite element program. Despite its simplicity, this method is applicable only for low variability of the structural properties. However, the computational efficiency is a major advantage to consider it for large problems where variability is low.

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Table IV. The results for shear forces and bending moments of bridge deck for random material and geometrical properties.

		Case A		Case B		Case C	
		PSFEM	MCS	PSFEM	MCS	PSFEM	MCS
Shear forces	Maximum shear force (kN)	24.390	23.578	24.435	23.472	25.392	23.468
	Average shear force (kN)	11.803	11.649	11.848	11.668	12.805	11.669
	Difference (According to MCS)	3.44%		4.10%		8.20%	
	Average difference (According to MCS)	1.42%		1.65%		5.05%	
	PSFEM average difference (According to Case A)			0.38%		8.5%	
	MCS average difference (According to Case A)			0.16%		0.17%	
Bending moments	Maximum bending moment (kNm)	140.981	137.592	140.996	137.023	141.001	137.026
	Average bending moment (kNm)	66.490	65.627	66.505	65.570	66.510	65.584
	Difference (According to MCS)	2.57%		3.10%		3.10%	
	Average difference (According to MCS)	1.36%		1.52%		1.52%	
	PSFEM average difference (According to Case A)			0.02%		0.03%	
	MCS average difference (According to Case B)	0.09%				0.02%	

It is seen also from the analysis results obtained from Cases A–C that dynamic response values for random cross-sectional area (Case C) are generally greater than those obtained from random elastic module and mass density for chosen systems.

4. CONCLUSIONS

In this paper, an efficient and accurate algorithm has been proposed to perform the probabilistic dynamic analysis of steel–concrete composite systems. Uncertainties in the structural parameters can be taken into account in this algorithm. The efficiency and accuracy of the proposed algorithm are validated by comparison with results of MCS. The comparison of the perturbation-based SFEM and MCS method for stochastic dynamic responses of structural systems made of composite sections subjected to ground motion is performed with two examples. The effects of various parameters on the dynamic responses of the composite frame and bridge are investigated, and the following conclusions can be drawn.

These numerical results of two examples clearly indicate a clear assessment of the stochastic dynamic responses of the proposed method in this paper and other analysis procedures. Although two methods agree very well and are evidently comparable in accuracy, PSFEM proposed herein is the most efficient and economical numerical solution procedure for the dynamic analysis of random variable material and geometrical properties and may be widely applied to the analysis of random structural dynamics problems.

These numerical conclusions show that displacements and internal forces are close to all random variables (elastic module, mass density and cross-sectional area) for PSFEM and MCS method. The dynamic response values obtained for the random variable cross-sectional area are generally higher than those of the other random variables for chosen composite systems.

The accuracy of the obtained results compared with those obtained from MCS method solution is remarkable. However, as the number of degrees of freedom of the structure and the number of uncertain parameters increase, the structural analyses based on MCS becomes very heavy from a computational point of view, and, in some cases, the computational effort makes them inapplicable.

The presented numerical technique is well suited for computer-aided analysis for structural systems made of steel–concrete composite sections. The PSFEM is suited for problems with low variability of the structural parameters. It was seen that this technique is very suitable for chosen COV value ($\alpha = 0.10$). It should be mentioned that the approach proposed should turn out useful in generating material data for the efficient SFEM analysis of various composites because of its relatively low computational cost it should also find applications.

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