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Fast practical analytical model for analysis of backfill-rectangular tank-fluid interaction systems

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ABSTRACT

The paper describes a simple, efficient and reasonably accurate analytical model for analysis of backfill-rectangular tank–fluid interaction systems. The presented model uses Housner's two-mass approximation for fluid interaction and mass-spring-dashpot system for backfill interaction and accommodates a variety of features that may affect the behavior of rectangular tanks including fluid volume, backfill geometry, soil properties and flexible/rigid wall types. Unlike the conventional finite element and the finite difference models that require considerable effort and knowledge to prepare the input data, the proposed model requires only a few pieces of data to define the problem and control the analysis. A series of finite element simulations were also fulfilled on two real rectangular tanks subjected to backfill and fluid effects. A reasonably good accord was obtained comparing the analytical predictions to results from numerical simulations. Thus, it can be stated that the model may be used effectively to perform a broad suite of parametric studies at the design stage and also as a reliable tool for estimating the system behavior. The results obtained from parametric seismic analyses indicated that backfill interaction, wall flexibility and fluid interaction considerably affect the lateral displacements. The sloshing response, however, was not practically affected by the backfill interaction and the wall flexibility.

1. Introduction

Lifeline systems are an integral part of infrastructures of cities and have vital importance for almost all sectors of the urban communities. The engineering community has long worked for building safe and reliable lifeline systems because their function is especially significant in assisting rapid recovery immediately following natural disasters. Furthermore, liquid storage tanks are also critical lifeline structures that are geographically dispersed over broad areas. After an earthquake, damage to the tanks may result in more serious disasters such as a secondary fatal effect to the modern cities. Damages to or collapse of the liquid storage tanks causes undesired effects such as shortage of drinking water and/or service liquid, uncontrolled fires and leakage or spillage of dangerous chemical liquids and liquefied gases [1]. Uncontrolled fires and spillage of dangerous liquids subsequent to a major earthquake may even cause more damages than the earthquake itself [2]. In that vein, the major problems related to the urgent water requirements and dramatic environmental scenes experienced just after the Japan earthquake occurred on March 11, 2011 have revealed the significance of these lifeline systems once again. Hence, all reasons mentioned above show that it is necessary to study the seismic behavior and design of tank systems for the requirements of earthquake hazard mitigation.

Problems associated with the seismic behavior of liquid storage tanks have shown that the soil–structure interaction (SSI) and fluid–structure interaction (FSI) have generally considerable influences on representing the response and design of the tanks [3]. Especially for a tank subjected to backfill effects, both effects must be strictly assessed. Damage to rectangular tanks can be great due to an incomplete understanding of the complex backfill and fluid interactions occurring during an earthquake.

More extensive research on SSI and FSI has addressed the application of modal analysis techniques [4]. The modal analysis is generally refered to as a more appropriate way regarding structural dynamic problems such as those associated with earthquake loading because it provides physical insights into which vibration modes contribute to the system behavior and only the lowest several modes are mostly needed to calculate responses in a majority of analyses. A key step in the current methods of dynamic analysis of the soil–structure–fluid system is to estimate modal characteristics which depend on stiffness and mass of the components composing the interaction system by using analytical, numerical and experimental methods.

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The seismic responses of various types of tanks have been examined by a number of researchers from the beginning of the twentieth century to the present. However, the bulk of these studies have concentrated on the ground-level cylindrical tanks and there are relatively few investigations focusing on the dynamic behavior of rectangular tanks. General reviews of chronological developments on the dynamic analysis of rectangular tanks can be found in the study published by Livaoglu [5]. As for other recent studies, Chen and Kianoush [6] used a procedure called the sequential method to calculate the dynamic response of rectangular tanks considering the effects of wall flexibility. In another study, Kianoush and Chen [7] investigated the response of rectangular tanks subjected to vertical ground acceleration. Chen and Kianoush [8] also presented a simplified method using the generalized single degree of freedom system for seismic analysis and design of rectangular tanks. Examinations on rectangular tanks considering backfill-wall–fluid interaction were carried out recently by Cakir et al. [9], Livaoglu and Cakir [10], and Livaoglu et al. [11]. Moreover, Cakir [12] carried out a comprehensive investigation of the earthquake behaviors of backfill-rectangular tank–fluid–soil/foundation systems.

Seismic analysis of rectangular tanks is usually a difficult problem and it depends on a lot of specifications some of which are backfill soil properties, relative stiffness of the wall/soil system, wall fixity conditions, fluid effects, foundation stability, and characteristics of applied earthquake motions. It is a well-known fact that the effect of backfill soil pressure is of great importance for a number of problems. Discussion of all research work on seismic soil pressure is, however, extensive and beyond the scope of this study. Rather, only some milestones that have influenced the design practice are described below.

The investigations performed by various researchers on soil–wall systems were comprehensively described in the state-of-the-art study carried out by Nazarian and Hadjian [13]. It is worth stating here that most of the studies presented in the article are based on limit analysis and thus actually provide significant estimations for very intense earthquakes only, since the achievement of a limit state in the soil requires large displacements. However, under low intensity seismic actions, the wall displacements are not adequate to activate the limit shear strength of the soil. In such cases, the dynamic pressure increment on the wall can still be evaluated considering a linear elastic soil behavior [14]. Among these approaches, Wood [15] modeled the wall–soil system using linear, elastic, plane strain conditions. Arias et al. [16] provided a simple analytical solution on the basis of a modified shear beam model for the elastic backfill soil under an arbitrary horizontal dynamic excitation. Veletsos and Younan [17,18] presented a simple approximation, yet sufficiently accurate elastic analytic solution under the plain strain conditions for vertical rigid wall retaining soil with a semi-infinite and uniform viscoelastic layer of constant thickness. Veletsos and Younan [19], and Younan and Veletsos [20] also extended the examinations to the flexible retaining walls elastically constrained at their base. Having adopted a similar approach, Theodorakopoulos et al. [21,22] and Theodorakopoulos [23] investigated the dynamic response of a rigid cantilever wall retaining a semi-infinite, uniform, fully-saturated poroelastic layer of soil. Furthermore, Mylonakis et al. [24], as an alternative to the Mononobe–Okabe equations, proposed a stress plasticity solution for determining gravitational and earthquake-induced earth pressures on walls retaining cohesionless soil.

Elastic, plane strain solutions based on the finite element model were achieved in the frequency domain and with the help of absorbing boundaries or infinite elements by Navarro and Samartin [25], Siller et al. [26], Zhao and Valliaphan [27] and Zhao and Xu [28]. Furthermore, Elgamal et al. [29] conducted numerical simulations and full-scale vibration tests to determine dynamic characteristics of a wall–backfill system. Gazetas et al. [30], modeling the soil as both elastic and inelastic material, submitted a finite element modelling of the dynamic stresses imposed on a variety of retaining systems under short-duration and impulsive base excitation. Psarropoulos et al. [31] built up a general finite element solution for analyzing the distribution of dynamic earth pressures on rigid and flexible walls. Madabhushi and Zeng [32] carried out numerical simulation and centrifuge modeling to analyze the seismic response of a cantilever wall with dry and saturated backfills.

As the literature indicates, most of the investigations on dynamic behavior of rectangular tanks involved mainly analytical and/or numerical solutions. In modeling, the analysts seek to exclude superfluous details but include all essential features, so that analysis of the model may not be unnecessarily complicated and may provide results that can describe the actual problem with sufficient accuracy. In the field of geotechnical and structural engineering, the finite element method and analytical approaches have played complementary roles. Due to the complexity of finite element techniques, investigators have directed their attention toward faster and more simplified, but equally accurate solutions, using simple agents like springs and dashpots to account for not only soil-structure but also fluid-structure interactions. The finite element method is usually verified against analytical methods; numerical stability and convergence studies are conducted. Then, the method is extended to problems that would be extremely difficult or impossible to solve using analytical methods. Furthermore, in some cases, this extension is also corroborated by laboratory controlled and/or in-situ tests. When an elaborate idealization of the problem is made, the finite element and approximate analytical analyses can be meaningful and powerful. However, it should not be forgotten that the approximate analytical and numerical results are only responsive to idealized conditions.

Considering the literature investigations and above discussions, it can be clearly stated that there is a need to develop a simple approach that can practically take into consideration the backfill-rectangular tank–fluid interaction system altogether. When the codes [33–35] about the rectangular tanks and/or retaining walls are examined, we may also find out that the simplified analytical and/or specific numerical methods are unavailable regarding how the backfill interaction effects can be taken into consideration. Furthermore, experimental or in-situ measurements unfortunately have not been sufficient to substantiate either method of analysis. Therefore, the primary objective of the present study is to propose a fast, simple and reliable analytical model for backfill-rectangular tank–fluid interaction systems, in order to prove that the proposed analytical procedure may be convenient for determining the dynamic behaviour of such systems and to investigate the seismic response of them in a fast and practical manner.

2. Proposed analytical model for the backfill-rectangular tank–fluid system

The first step in the proposed method is to clarify the statement of the problem since the analysis of the backfill-rectangular tank–fluid system is a complex problem due to both the fluid and backfill interactions. The next step is to identify the factors or the variables of the problem, which can be defined as anything that affects the solution. Both the schematic representation of the problem under consideration and the mechanical model for the backfill-rectangular tank–fluid interaction system are shown in Fig. 1. In this proposed model, it is assumed that

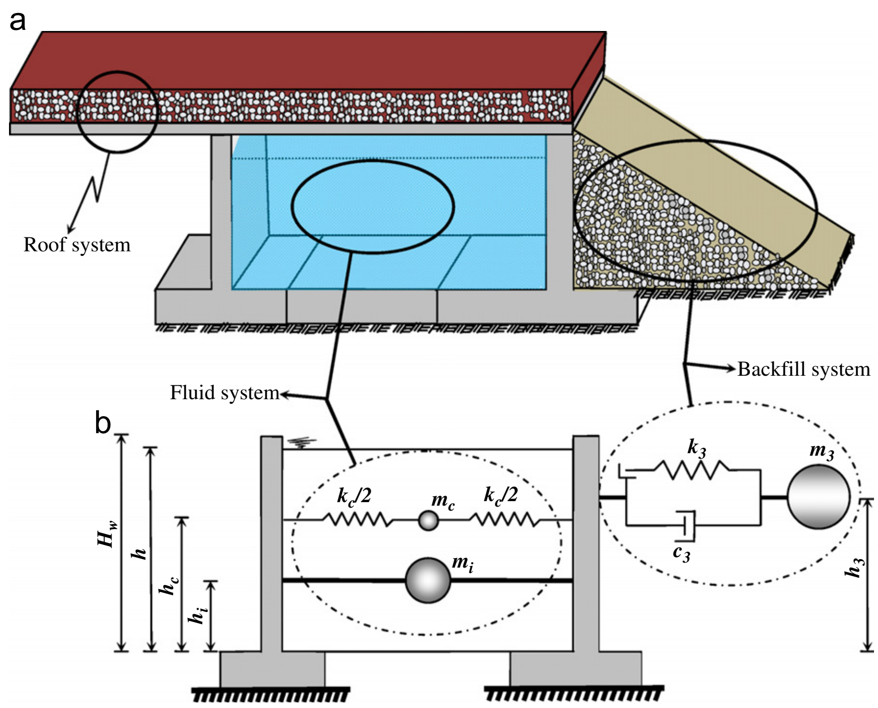

Fig. 1. (a) Scheme of the backfill-rectangular tank–fluid system investigated, (b) proposed mechanical model for the backfill-rectangular tank–fluid interaction system.

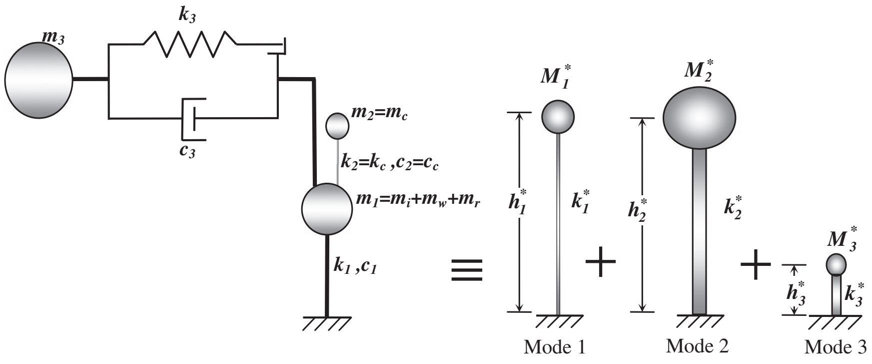

Fig. 2. The mathematical model and modal representation of the backfill-exterior wall–fluid system.

both structure and soil medium have elastic and homogeneous material properties. The foundation of the wall is assumed as rigidly fixed to the soil medium. Through the interface between wall and backfill, only lateral movement is considered and other movements and rotations are ignored. It is worth mentioning here that this method should not be used to analyze for fully embedded tanks. The proposed model is for the fluid–tank wall–backfill system in which backfill is generally designed by using granular material to cover tank wall as seen from Fig. 1. For the embedded case, in which the tank wall is subjected to dynamic load arisen from the half soil space, the frequency dependent damping and stiffness properties must be included for estimation.

Fluid–structure interaction can be modeled using simplified analytical methods such as two-mass representation [36], multi mass representation [37] or the method provided in Eurocode-8 [34] during analytical investigations. When all approaches cited above are compared with each other, one can clearly see that all of them lead to similar results because they share the same theoretical background. Therefore, Housner's two-mass representation is considered to model fluid-wall interaction in this study since it has been widely used in the literature and/or recommended by current major earthquake codes. Housner's [36] simplified analysis procedure takes into account the fluid which is separated into an impulsive mass m_i that is rigidly connected with the tank walls and a convective mass m_c that is attached to the walls using stiffness of spring (k_c). The heights of the convective mass (h_c) and impulsive mass (h_i), depending on depth of the fluid h, are also provided by Housner [36]. The elaborate explanations on calculations of these parameters can be found for rectangular tanks and cylindrical tanks in Livaoglu [5] and Livaoglu and Dogangun [38], respectively. Moreover, as seen in Fig. 1, the backfill–wall interaction is modeled by means of the mass-spring-dashpot system, which is connected to the tank container wall at a height of h_3 that is equal to $(2/\pi)H_w=0.637H_w$ where H_w is the height of the wall. To obtain a simplified model, the approach is based on the simplifying assumption that no de-bonding or relative slip is allowed to occur at the wall–soil interface.

In light of the above arguments, the mathematical model and modal representation of the backfill-exterior wall–fluid interaction system are depicted in Fig. 2.

To obtain a solution, the concerned design parameters as stifnesses and masses, must be determined primarily. The mass

m_1 consists of the summation of impulsive mass (m_i), mass of wall (m_w) and effective mass of the roof (m_r) which is the corresponding part of the roof mass acting on the exterior wall of the rectangular tank except for those carried by columns and the interior wall. The mass m_2 is directly equal to convective mass (m_c) and the mass m_3 refers to backfill mass. The lateral stiffness of the exterior wall, k_1, can readily be determined as $k_1 = 12EI_{\mathrm{ort}}/H_w^3$ since the roof slab is rigid. The stiffness k_2 is directly equal to convective stiffness (k_c). The stiffness k_3 is the average shear stiffness for backfill soil, which may conveniently be expressed as the product of the shear modulus of backfill (G) and the reduced cross sectional area (F') of backfill. The reduced cross sectional area can be estimated as $F' = F/k'$ where F is the cross sectional area of backfill at the average level and k' is a coefficient which can be taken as 1.2 for a rectangular cross sectional area. The parameters of c_1, c_2, c_3 are the damping values for impulsive mode, convective mode and backfill soil, respectively.

The development of the simplified analytical solution may be derived from a physical interpretation of the solution to the differential equation. Considering dynamic equilibrium, from Fig. 2, the following equations can be written.

$$m_1\ddot{u}_1 + c_1\dot{u}_1 + k_1u_1 + c_2(\dot{u}_1 - \dot{u}_2) + k_2(u_1 - u_2) + c_3(\dot{u}_1 - \dot{u}_3) + k_3(u_1 - u_3) = P_1(t) \quad (1)$$

$$m_2\ddot{u}_2 + c_2(\dot{u}_2 - \dot{u}_1) + k_2(u_2 - u_1) = P_2(t) \quad (2)$$

$$m_3\ddot{u}_3 + c_3(\dot{u}_3 - \dot{u}_1) + k_3(u_3 - u_1) = P_3(t) \quad (3)$$

Combining Eqs. (1)–(3), basic dynamic equations can be written in matrix form:

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} \begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \\ \ddot{u}_3 \end{Bmatrix} + \begin{bmatrix} c_1+c_2+c_3 & -c_2 & -c_3 \\ -c_2 & c_2 & 0 \\ -c_3 & 0 & c_3 \end{bmatrix} \begin{Bmatrix} \dot{u}_1 \\ \dot{u}_2 \\ \dot{u}_3 \end{Bmatrix} + \begin{bmatrix} k_1+k_2+k_3 & -k_2 & -k_3 \\ -k_2 & k_2 & 0 \\ -k_3 & 0 & k_3 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} P_1(t) \\ P_2(t) \\ P_3(t) \end{Bmatrix} \quad (4)$$

where (u_1, u_2, u_3), ($\dot{u}_1$, $\dot{u}_2$, $\dot{u}_3$), ($\ddot{u}_1$, $\ddot{u}_2$, $\ddot{u}_3$) are the displacements, velocities and accelerations of masses m_1, m_2, m_3, respectively, and $P_1(t)$, $P_2(t)$, $P_3(t)$are the applied external forces. It is worth noting that because the natural frequencies of the system in the analytical modal analysis are determined by using undamped free vibration equation of motions which can, in turn, be derived from the preceding equations, the data regarding both the damping matrix and the external forces are not discussed herein. However, these data will be elucidated later in the section on seismic analysis of backfill-rectangular tank–fluid system.

The obtained equations can be solved by utilizing the modal analysis techniques. The modal properties such as effective modal masses (M_1^*,M_2^*,M_3^*), heights (h_1^*,h_2^*,h_3^*) and stiffnesses (k_1^*,k_2^*,k_3^*) should be determined (see Fig. 2). These modal properties can be estimated using Eqs. (5) and (6) [39]. In this study, the modal analyses are conducted assuming elastic material properties.

$$M_n^* = \Gamma_n L_n^h = \frac{(L_n^h)}{M_n}; \quad h_n^* = \frac{L_n^\theta}{L_n^h}; \quad k_n^* = \omega_n^2 M_n^* \quad (5)$$

where;

$$M_n = \phi_n^T m \phi_n = \sum_{j=1}^{N} m_j \phi_{jn}^2; \quad \Gamma_n = \frac{L_n^h}{M_n};$$
$$L_n^h = \sum_{j=1}^{N} m_j \phi_{jn}; \quad L_n^\theta = \sum_{j=1}^{N} h_j m_j \phi_{jn} \quad (6)$$

where N, ϕ_n and ω_n^2 are the total mode number, the **n**th mode vector and its eigenvalue, respectively. Due to the absolute differences between the sloshing stiffness and the stiffness of the supporting system, it can be stated that the first mode represents the convective mode, and the second one represents the impulsive mode. Furthermore, the backfill behavior is represented by the third mode.

3. Numerical application

This section provides a brief description of the interaction systems under consideration. In the following sections, the details of finite element models of the systems developed are presented.

3.1. Description of the rectangular tank systems examined

To demonstrate the proposed methodology, a reinforced concrete rectangular (prismatic) tank with a container capacity of 4000 m^3 (TANK_1) and a rectangular tank with a capacity of 15,000 m^3 (TANK_2) are investigated. The general and structural properties of these two tank systems that have typical layout for the tanks mostly built in Turkey are presented below.

3.1.1. General properties of TANK_1

The rectangular tank examined was constructed in Hisar area of Bolu, Turkey in 1978. The tank had partly suffered from non-structural damage during the 1999 Düzce earthquake, but no structural damage occurred, and only some repairs were made in the components including mortar, plaster and ceramic etc. The rectangular tank under consideration has two main divisions. Based on the in-situ investigations of the authors, it was determined that the roof of the tank was constructed as a two-way beam slab supported by 6 m-height-slender columns which have 0.3 × 0.3 m plan geometry, and the exterior walls of the tank had a constant thickness of 0.3 m. It was also ascertained that both the roof slab and covering had a thickness of 0.15 m. Furthermore, the depth of water within the container was measured as 5 m. The other geometrical characteristics defined by the authors including the foundation system and the side and top views of the rectangular tank are shown in Fig. 3. The concrete strength was measured as 14 MPa using the Schmidt Test Hammer in accordance with ASTM C805. The Young's modulus, Poisson's ratio, and unit weight of the concrete were interpreted as 26,160 MPa, 0.2 and 25 $\mathrm{kN/m}^3$, respectively. In addition, after representative soil samples were taken from the field, they were tested in the laboratory, and it was determined that the backfill soil could be classified as silty sand, poorly graded gravel-sand-silt mixtures. Thus, examining the mechanical properties recommended in the literature for the abovementioned soil class, Young's Modulus, Poisson's ratio and the unit weight of the soil were taken to be 20 MPa, 0.3 and 19 $\mathrm{kN/m}^3$, respectively. Furthermore, the internal friction angle for the backfill soil was considered as $\phi = 25^\circ$. According to the investigation carried out by Livaoglu et al. [11], when the internal friction angle increases from 25° to 40°, the maximum lateral displacements are not considerably affected. Thus, from the engineering point of view, variation of internal friction angle of backfill can be accepted with no influence for seismic analysis of backfill-rectangular tank–fluid system.

3.1.2. General properties of TANK_2

The other rectangular tank taken into consideration in this study was constructed in the Atakum area of Samsun, Turkey in 1995. The tank has two main divisions. According to in-situ investigations of authors and their application projects reviews, it was determined that the roof of the tank was constructed as a slab supported by 6 m-height columns which have 0.4 × 0.4 m plan geometry. It was also ascertained that the roof slab and

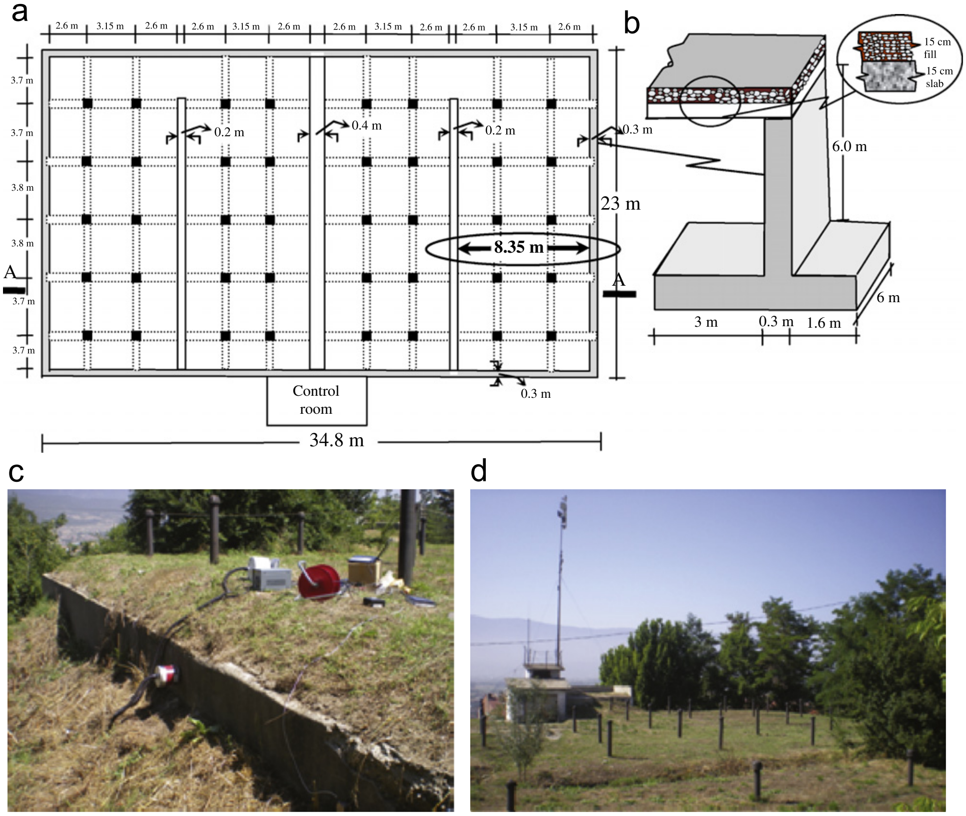

Fig. 3. Geometrical properties and images of TANK_1 (a) plan, (b) elevation, (c) side view and (d) top view.

covering had the thicknesses of 0.20 and 0.60 m, respectively. Furthermore, the height of the exterior wall is 6 m and the depth of water within the container was measured as 4 m during in-situ tests. The other geometrical characteristics determined by the authors including the foundation system and the top and side views of the tank are depicted in Fig. 4. The concrete strength was estimated as 18 MPa through the instrumentality of the Schmidt Test Hammer in accordance with ASTM C805. The Young's modulus, Poisson's ratio and unit weight of the concrete were calculated as 28,000 MPa, 0.2 and 25 kN/m^3, respectively. In addition, after representative soil samples taken from the field were tested in the laboratory, it was determined that the backfill soil could be classified as silty sand. Thus, having examined the mechanical properties recommended in the literature for silty sand soil, we found out Young's Modulus, Poisson's ratio and the unit weight of the soil to be 20 MPa, 0.3 and 19 kN/m^3, respectively. Moreover, similar to the other case, the internal friction angle of $\phi = 25^\circ$ for the backfill soil was taken into account.

3.2. Finite element analyses for backfill-rectangular tank–fluid system

It is not possible to obtain a two-dimensional representation that will approximate both the dynamic stiffness and damping over a reasonable range of frequencies [40]. In general, the two-dimensional models overestimate the damping associated with the three-dimensional problem. Furthermore, it was found that a proper selection of the two-dimensional model make it possible to obtain close approximations to the system frequencies. Since the dampings associated with the low frequency modes are overestimated, the earthquake response of the structure, obtained by the two-dimensional model, is underestimated. In the light of this information, the 3D analysis of the interaction system is considered in this study. Through analysis of all cases, the backfill-rectangular tank–fluid interaction models were implemented by means of the finite element software ANSYS [41]. The interaction models proposed are refered to TANK_1 and TANK_2 systems as presented in Fig. 5. In these models, the structural wall is modeled with solid elements defined by eight nodes having three degrees-of-freedom at each node: translations in the nodal x, y, z directions. It is worth mentioning here that depending on the dimensions of the wall, these elements were used as either a single row or a double row. On the other hand, when the lateral response of the tank wall is investigated, it can be observed that the wall behaves laterally as a rigid thick plate due to the adjacent backfill and its very thick bottom dimension. Additionally, it must be stated here that the parametric study was conducted in order to decide the most appropriate and efficient mesh, prior to adopting the optimal mesh of the model. The roof system is modeled with quadrilateral shell elements defined by four nodes having six degrees-of-freedom at each node: translations in the nodal x, y, z directions and rotations about the nodal x, y, z axes and also with added mass for roof covering. Actually, despite its structural simplicity, the dynamic response of rectangular tanks is part of a rather complex dynamic system. What makes that response so complicated is the dynamic interaction between both the tank wall and backfill soil, and the tank wall and fluid. Therefore, the sophisticated interaction system must be reasonably modelled. The modeling of the wall–backfill interaction also requires special handling of interface elements between the wall and adjacent soil. Thus, to model backfill–wall interaction, as a special interface element, the unidirectional element with nonlinear generalized force-deflection capability is regulated representing very rigid compression characteristics with tensionless in the interaction face of the backfill–wall system. The element has longitudinal or torsional capability in 1-D, 2-D, or 3-D applications. The longitudinal option is a uniaxial tension–compression element with up to three degrees of freedom at each node:

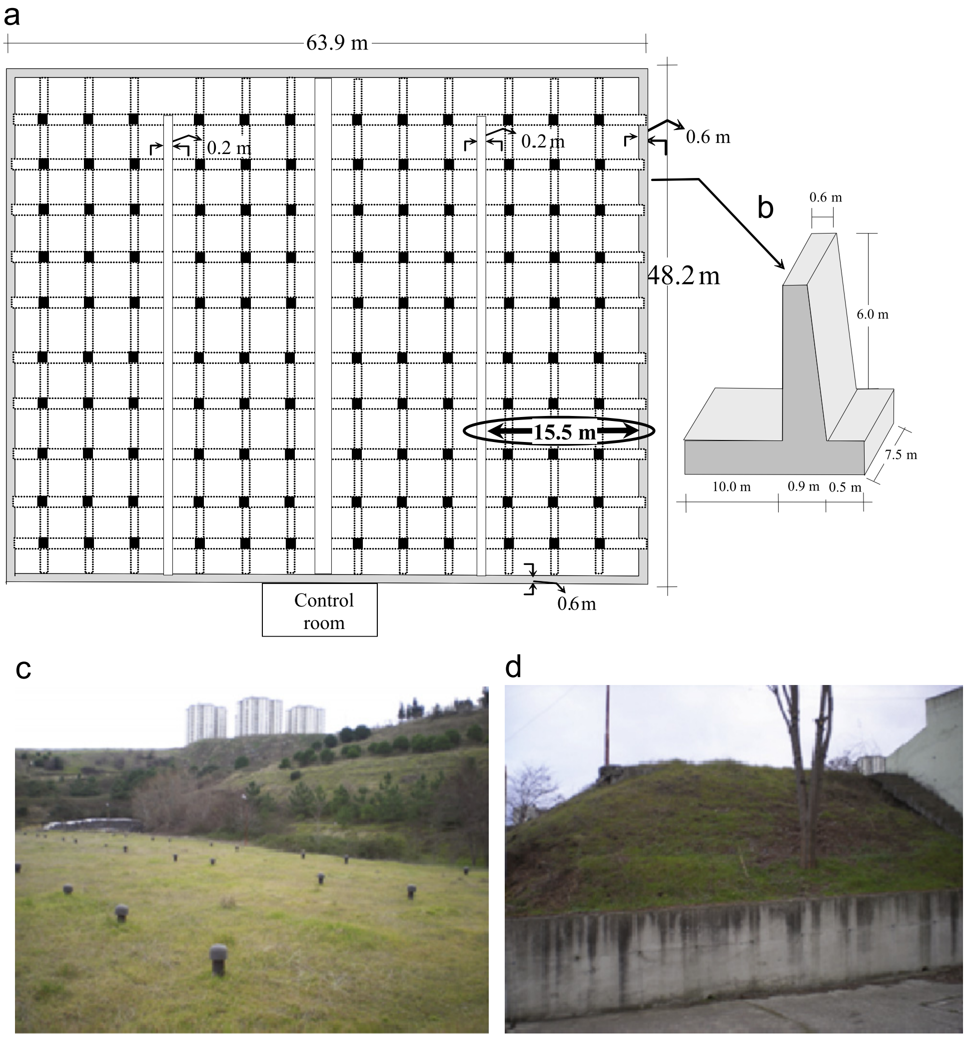

Fig. 4. Geometrical properties and images of TANK_2 (a) plan, (b) elevation, (c) top view and (d) side view.

translations in the nodal *x*, *y*, and *z* directions. The 1-D longitudinal option in the direction of normal to the tank wall is considered to simulate the backfill-tank interaction surface.

Fluid-structure interaction can be modeled in different ways in numerical modelling. The well-known ways include added mass, Lagrangian, Eulerian and Lagrangian–Eulerian approaches. A displacement-based Lagrangian approach, including the effect of fluid–wall interaction, is adopted in this study. The fluid element, defined by eight nodes having three translational degrees-of-freedom at each node, is specially formulated to model fluid contained within the container having no net flow rate. Furthermore, the simulation of the infinite medium in the numerical modeling of dynamic soil–structure interaction problems is extremely crucial. The appropriate approximations for such a typical problem investigated in this study can be executed utilizing the artificial and/or transmitting boundaries in order to not only avoid reflection but also consider the radiation effects of the propagating waves from the interfaces on the extracted cross-section of backfill. There are different types of boundaries available in the literature in frequency and time domains with different sensitivities. In this study, the viscous boundary model developed by Lysmer and Kuhlemeyer [42] is used for three dimensions to consider radiational effect of the seismic waves through the soil medium. In other words, we meant to avoid the box effect in the direction of perpendicular to the excited or normal direction of the wall. Mathematical details of modeling of fluid and bounded media can be found in another study of the author [43]. In addition, elastoplastic behavior of backfill soil is described by well-known Drucker–Prager yield criteria.

4. Results and discussions

In this section, the results obtained from analytical and numerical models are presented. Furthermore, the comparisons of the results of analytical investigations and numerical simulations are provided in this section.

4.1. Analytical results

This section reports the results of analyses of the analytical model with fixed base assumptions followed by discussion of the findings. The simplified model of backfill-rectangular tank–fluid system for TANK_1 is depicted in Fig. 6(a). Through using the proposed analytical model, the values of mass and stifness, which are necessary to compute the natural frequencies of the interaction system, are presented in Fig. 6(b). The modal characteristics such as the effective modal masses, stifnesses and modal frequencies can be seen in Fig. 6(c).

As the above figures demonstrate, the mode frequencies were calculated as 0.299, 2.23 and 8.05 Hz, respectively. As pointed out earlier, due to the absolute differences between the sloshing

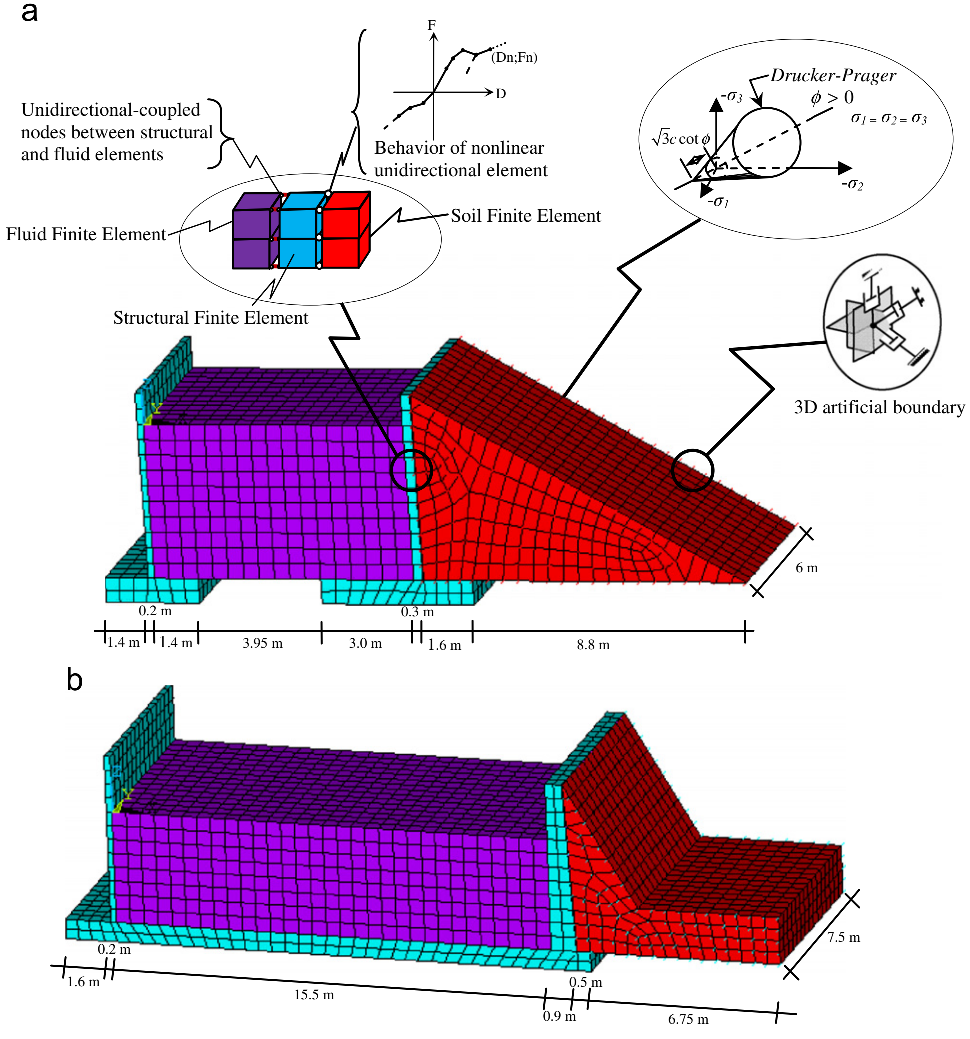

Fig. 5. Proposed finite element models for (a) TANK_1 system and (b) TANK_2 system.

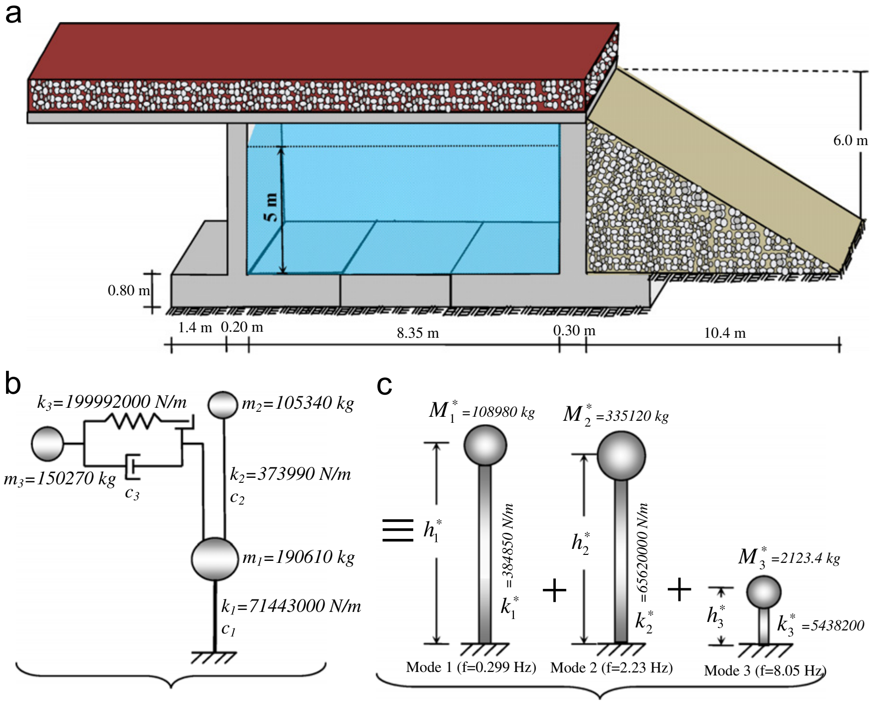

Fig. 6. (a) The simplified model of the backfill-rectangular tank–fluid system for TANK_1. (b) The stiffness and mass values computed from the analysis of the backfill-exterior wall–fluid interaction system. (c) The modal characteristics estimated from analysis of the backfill-exterior wall–fluid interaction system.

stiffness and the stiffness of the supporting system, it is worth saying here that the first and second modes represent the convective and impulsive modes, respectively. The sum of the effective modal masses reaches, 99% of the total mass, except for the convective mass, is represented by the second mode, and the remaining mass is represented by the third mode. On the other hand, 42% of the total water mass is represented by the convective mode, and 61% of it is represented by the impulsive mode. It means that the modes considered in the analysis are enough to represent all the system behaviour.

Similarly, the simplified model for TANK_2 is illustrated in Fig. 7(a). Furthermore, mass and stiffness values as well as the modal characteristics of the interaction system are given in Fig. 7(b) and (c), respectively.

As shown in Fig. 7(c), the mode frequencies were computed as 0.185, 5.52 and 12.96 Hz, respectively. It can be clearly seen that 42% of the total mass, except for the convective mass, is represented by the second mode, and the remaining mass is represented by the third mode. Furthermore, 69% of the total water mass is represented by the convective component, and approximately 30% of it is represented by the impulsive component.

4.2. Numerical results

Figs. 8 and 9 show the mode shapes of the finite element models with fixed base assumption for TANK_1 and TANK_2, respectively. Only the first three vibration modes, which have the ability to represent all system behaviour based on mass participation ratios or effective modal masses, were identified for both tank systems. The statement of “first three vibration modes” does not mean these modes are ranged on the first order through the all modes according to the frequency results. These are ranged due to its effective masses. While the frequencies of the first three modes given in Fig. 8 were estimated as 2.325, 5.411 and 6.513 Hz for TANK_1, the same quantities were computed as 5.24, 6.24 and 7.51 Hz for TANK_2 (Fig. 9). It is necessary to emphasize here that the mode shapes given in Figs. 8 and 9 both belong to the structural systems, including backfill as well as fluid

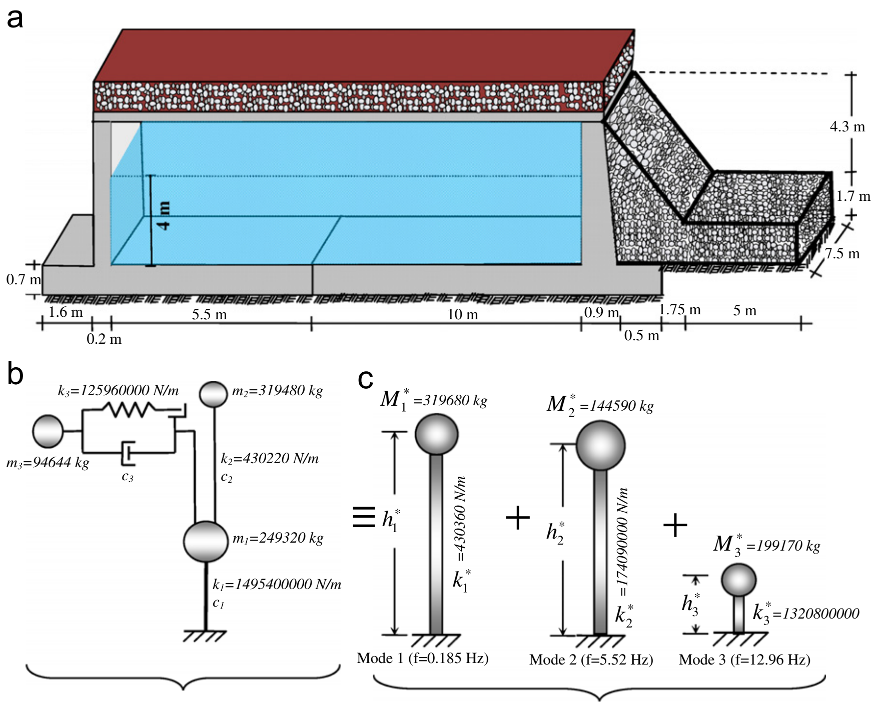

Fig. 7. (a) The simplified model of the backfill-rectangular tank–fluid system for TANK_2. (b) The stiffness and mass values computed from the analysis of the backfill-exterior wall–fluid interaction system. (c) The modal characteristics estimated from analysis of the backfill-exterior wall–fluid interaction system.

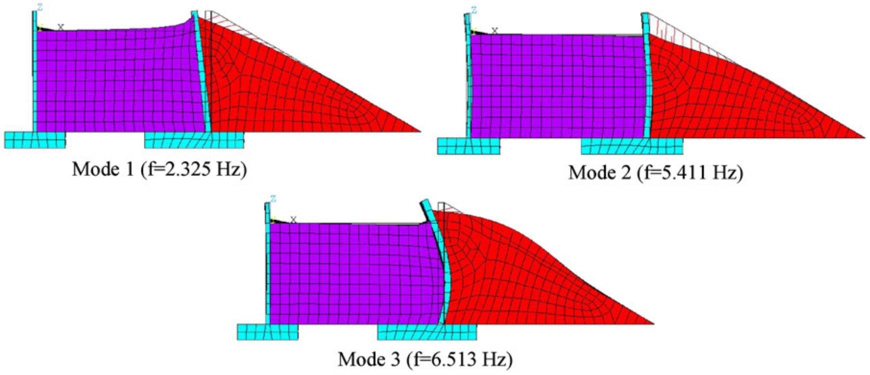

Fig. 8. First three mode shapes and frequencies for TANK_1.

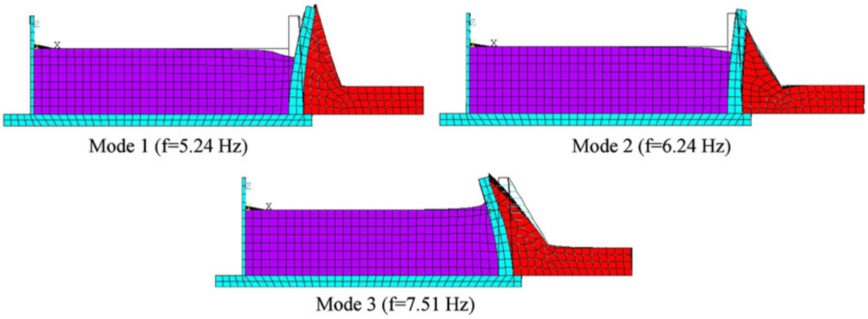
Mode 1 (f=5.24 Hz)
Mode 2 (f=6.24 Hz)
Mode 3 (f=7.51 Hz)

Fig. 9. First three mode shapes and frequencies for TANK_2.

Table 1
Comparison of analytical and numerical results for TANK_1 and TANK_2.

	Mode categories	Mode descriptions	Frequencies (Hz)		Error (%)
			Analytical model	Numerical model	
TANK_1	Fluid	Sloshing mode	0.299	–	–
	Structure	First mode	2.23	2.325	4
		Second mode	–	5.411	–
		Third mode	–	6.513	–
	Backfill	Backfill mode	8.05	–	–
TANK_2	Fluid	Sloshing mode	0.185	–	–
	Structure	First mode	5.52	5.24	5
		Second mode	–	6.24	–
		Third mode	–	7.51	–
	Backfill	Backfill mode	12.96	–	–

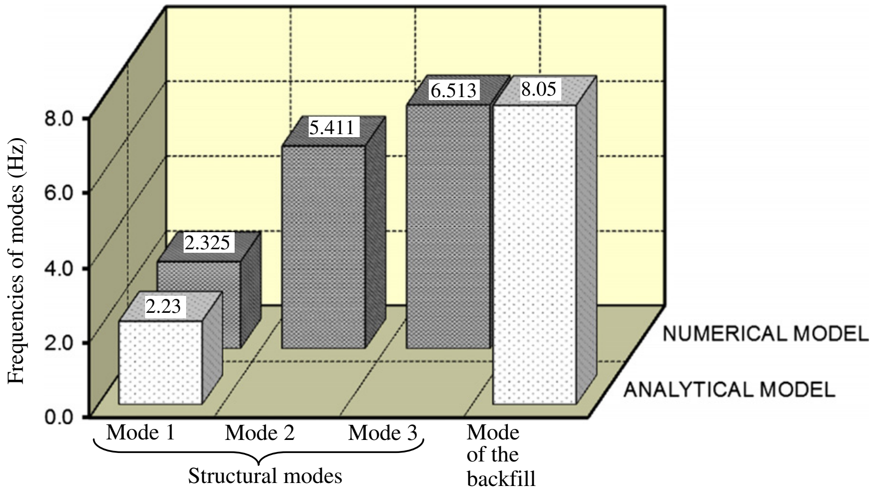

Fig. 10. The mode frequencies of TANK_1 according to the proposed models.

interaction effects. The fluid sloshing modes in these figures are not illustrated.

4.3. General comparison of results

The natural frequencies predicted by a three-dimensional finite element model developed by using ANSYS, the proposed analytical model are presented in Table 1 for TANK_1. Fig. 10 also includes the comparison of the mode frequencies of the TANK_1 system. As Table 1 depicts, the system behavior is represented by only three modes in the analytical model. However, the comparison of the modes related to the sloshing and backfill soil were not given since only the modes related to the structure subjected to backfill and fluid interaction effects were investigated. When a comparison is made for the first mode, it is seen that the frequency value obtained from the analytical model is in good harmony with the result obtained from numerical model so that the error is approximately 4%.

Similarly, the computed values of frequency are compared with each other in Table 1 and Fig. 11 for the TANK_2 system. As shown, the analytical and numerical predictions of natural frequency agree within about 5%. Indeed, these reflect successful predictions knowing that there can be some uncertainties and difficulties encountered in the approximations and drawbacks in the proposed models. Therefore, it can be pointed out that in the types of structures under investigation, the calculated errors are negligible from the engineering point of view due to such a complex interacting phenomena and the inherent variability and uncertainties of soil properties.

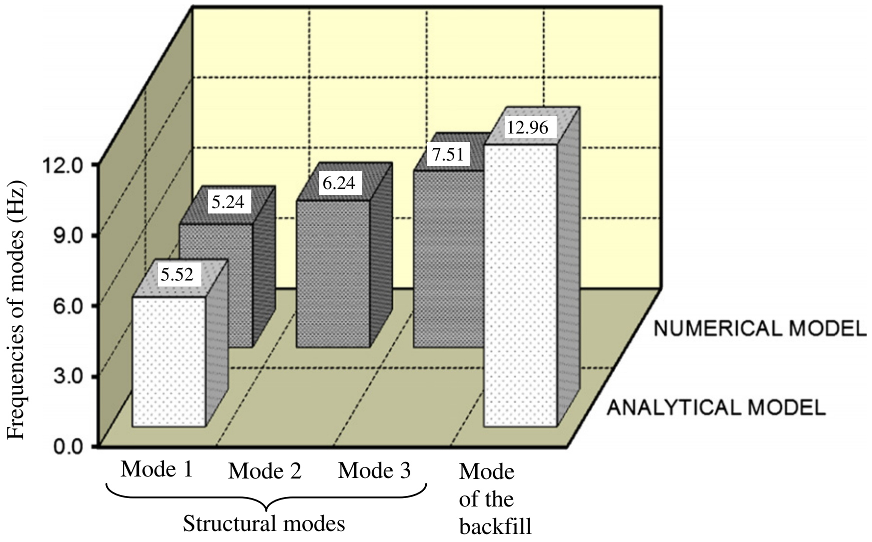

Fig. 11. The mode frequencies of TANK_2 according to the proposed models.

5. Seismic analysis of backfill-rectangular tank–fluid system

The transient analysis using Newmark's method [44] was carried out for seismic analysis of backfill-rectangular tank–fluid system. The TANK_2 system addressed previously was selected as the reference structure. The response of the interaction system was analyzed in case to be induced by the first 32 s of the East–West component of the strong-ground motion recorded during November 12, 1999 Düzce Earthquake in Düzce Meteorology Station, Turkey. The time step $\Delta t = 0.005$ s was chosen to be small enough to determine accurately the response of the system with natural periods T_n, the shortest of which is 0.0772 s. Furthermore, Rayleigh damping was utilized for constituting the damping matrix. The damping values for rectangular tanks were taken as 5% for the impulsive component and 0.5% for the convective component as recommended in most literature.

A number of seismic analyses with variation of parameters such as backfill soil type, wall flexibility and fluid height were conducted using the suggested analytical procedure. To illustrate the effects of the variation of the dynamic behavior of rectangular tank depending on different soil conditions, seven different backfill soil properties are named from S1 to S7. The Poisson's ratio (ν) and the unit weight (γ) of the considered soil types are equal to 0.3 and 19 kN/m^3, respectively. The Young modules (E) of the considered soil types which are named between S1 to S7 in increasing order are equal to 2.5, 5, 10, 20, 75, 150 and 300 MPa, respectively. The rectangular tanks with three different wall thicknesses were also considered to evaluate the wall flexibility effects. The first one was named as flexible tank (Case_1) having a variable wall thickness ranging from 0.4 m at the top to 0.6 m at the bottom. The second one was named as rigid tank (Case_2) which is the current case of the TANK_2. The last one was named as very rigid tank (Case_3) having a variable wall thickness ranging from 0.9 m at the top to 1.1 m at the bottom. Moreover, five different fluid heights were also taken into account to determine the fluid interaction effects.

The analyses were carried out via a computer program coded for the seismic analysis of rectangular tanks. Accordingly, the lateral displacement response of the wall obtained at the height of the impulsive mass and the sloshing response of fluid were determined. The results of the lateral and sloshing displacements were given in Tables 2 and 3, respectively. Furthermore, the effects of the backfill interaction, wall flexibility and fluid interaction on seismic response were illustrated and discussed comparatively below.

Table 2
Maximum results of the wall displacements at the height of the impulsive mass and their occurrence times.

Soil types	Case_1 (Flexible tank)		Case_2 (Rigid tank)		Case_3 (Very rigid tank)	
	Displacements (u)		Displacements (u)		Displacements (u)	
	Time (s)	Value (m)	Time (s)	Value (m)	Time (s)	Value (m)
S1	11.165	−0.00338	11.195	−0.00099	3.435	−0.00044
S2	4.89	−0.00527	4.88	−0.00136	4.875	−0.00057
S3	3.9	0.00490	3.45	−0.00133	3.445	−0.00059
S4	3.43	−0.00813	3.395	−0.00134	3.41	−0.00055
S5	3.38	−0.00531	3.44	−0.00160	3.415	−0.00051
S6	3.92	0.00536	3.41	−0.00144	3.435	−0.00052
S7	3.915	0.00548	3.03	0.00137	3.425	−0.00054

Table 3
Maximum results of the sloshing displacements and their occurrence times.

Soil types	Case_1 (Flexible tank)		Case_2 (Rigid tank)		Case_3 (Very rigid tank)	
	Sloshing displacements (u_s)		Sloshing displacements (u_s)		Sloshing displacements (u_s)	
	Time (s)	Value (m)	Time (s)	Value (m)	Time (s)	Value (m)
S1	14.12	2.259	14.115	2.258	14.115	2.257
S2	14.12	2.259	14.115	2.258	14.115	2.257
S3	14.12	2.259	14.115	2.258	14.115	2.257
S4	14.12	2.259	14.115	2.258	14.115	2.257
S5	14.12	2.259	14.115	2.258	14.115	2.257
S6	14.12	2.259	14.115	2.258	14.115	2.257
S7	14.12	2.259	14.115	2.258	14.115	2.257

5.1. *Backfill interaction effects on the lateral displacement*

Using the proposed analytical procedure, it is possible to determine the displacement responses at the height of the impulsive mass depending on various backfill soil conditions. As previously stated, all maximum responses and their occurrence times were presented in Table 2. Table 2 indicates that the responses of the systems are different from each other so that the maximum values of displacements and their occurence times changed with changing soil conditions for both flexible and rigid tanks. Here, it has to be stated that since all results of the analyzed models cannot be illustrated here, some comparisons were selected to describe the system behaviour. In this context, the comparisons of time history responses of the selected systems are shown for Case_1 in Fig. 12(a), and for Case_2 in Fig. 12(b). The most significant point arising from these comparisons is that the variation of backfill soil stiffness notably affects the displacement response of the system. For example, as from Fig. 12(a) for Case_1 demonstrates, while the maximum lateral displacement was calculated as 0.00338 m at 11.165 s for S1 soil type, the same quantity was computed as 0.00813 m at 3.43 s for S4 soil type. Therefore, it can be maintained that backfill–wall interaction affects the system behavior so that the increment in the displacement response is almost at a level of 140% between the S1 and S4 soil types. If similar comparisons are made for Case_2 as in Fig. 12(b), similar trend and backfill interaction effect can be observed so that the increase in the displacement response is almost 62% between the S1 and S5 soil types. These variations reflect a significant backfill interaction influence on the response. When similar comparisons are made for Case_3, it can be clearly seen that the relatively soft soil conditions changed the displacement response more significantly compared with the relatively stiff soil conditions. For example, while the displacement response decreased approximately 25% from S3 to S1, the same quantity decreased only 5% from S7 to S5. Consequently, these comparisons confirmed that the exclusion of the accurate backfill properties may cause underestimation or overestimation of the displacement response, and this, in turn, highly affects the design process due to the displacement sensitivity of this type of structure.

5.2. Backfill interaction effects on the sloshing displacement

The maximum sloshing displacements and their occurrence times estimated through the analyzed models can be seen in Table 3. These results show that, as pointed out by Veletsos and Tang [45] and Livaoglu [5] for the laterally excited liquid storage tanks, soil–structure interaction may significantly change the

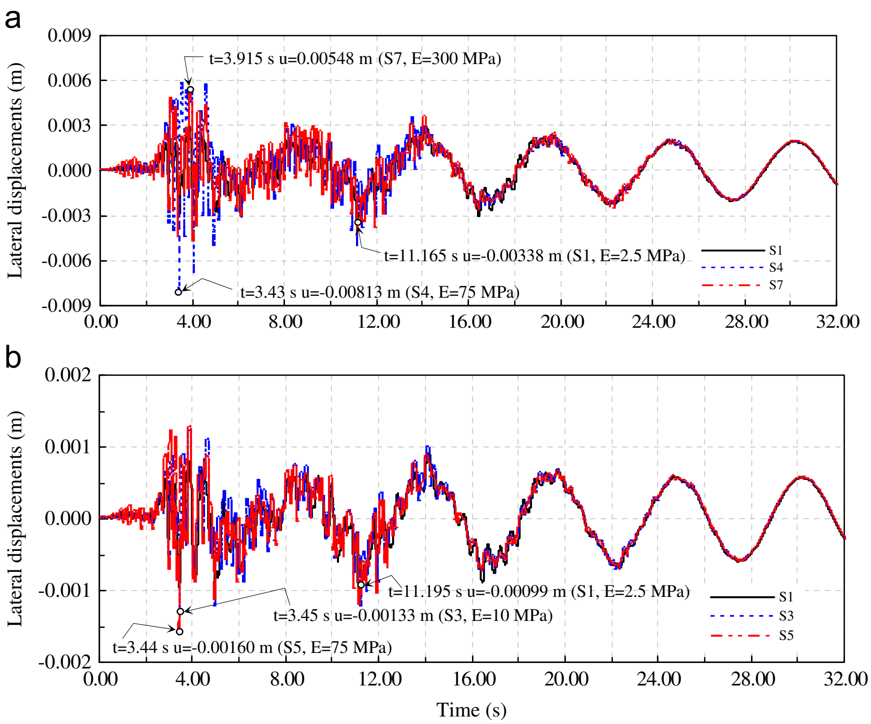

Fig. 12. Variations of the lateral displacements in time (a) for Case_1(flexible) with S1, S4 and S7 soil types and (b) for Case_2(rigid) with S1, S3 and S5 soil types.

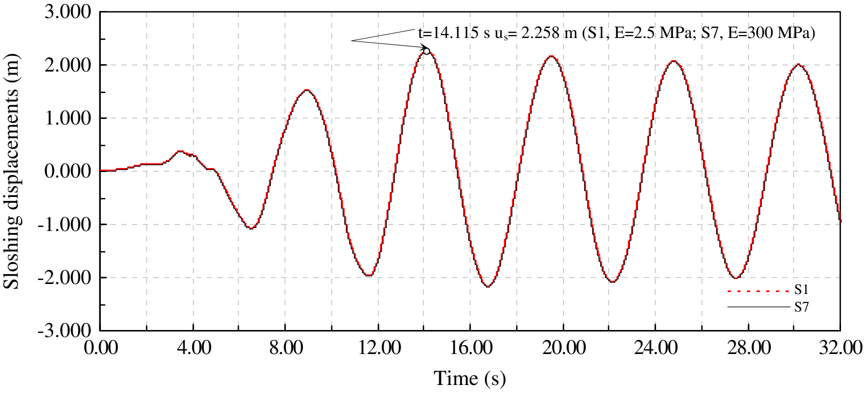

Fig.13. Variations of the sloshing displacements in time for Case_2 with S1 and S7 soil types.

impulsive component of response. However, it has a negligible effect on the convective component of the fluid within rectangular tanks. Based on the results, it is concluded that the maximum sloshing occurs approximately as 2.26 m at 14.115–14.12 s for all soil conditions. For instance, the time histories of sloshing responses for Case_2 with S1 and S7 soil types are presented in Fig. 13. As Fig. 13 depicts, in terms of the sloshing amplitude, the deviations between S1 and S7 soil types exhibited that the responses are almost coincided. Thus, we can state that the backfill interaction does not have any considerable effects on the sloshing response as already pointed out in the literature.

5.3. *Wall flexibility effects on the displacements*

The overall comparisons carried out previously indicate that the wall flexibility effects on displacements can be easily observed. The time histories of the lateral displacement responses for S4 and S6 soil types are illustrated in Fig. 14(a) and (b), respectively. As these figures indicate, the tank wall flexibility reflects a significant backfill interaction influence on the response. For example, while the maximum value of displacement was estimated as 0.00134 m for Case_2 (rigid tank) and computed as 0.00813 m for Case_1 (flexible tank), the response increased dramatically due to the flexibility of tank wall. When similar

Table 4
Maximum results of the wall displacements at the height of the impulsive mass and their occurrence times according to the fluid height.

Fluid height (h)(m)	Case_1 (Flexible tank)	
	Displacements (*u*)	
	Time (s)	Value (m)
1	3.415	−0.00435
2	3.415	−0.00493
3	3.42	−0.00608
4	3.43	−0.00813
5	3.445	−0.01068

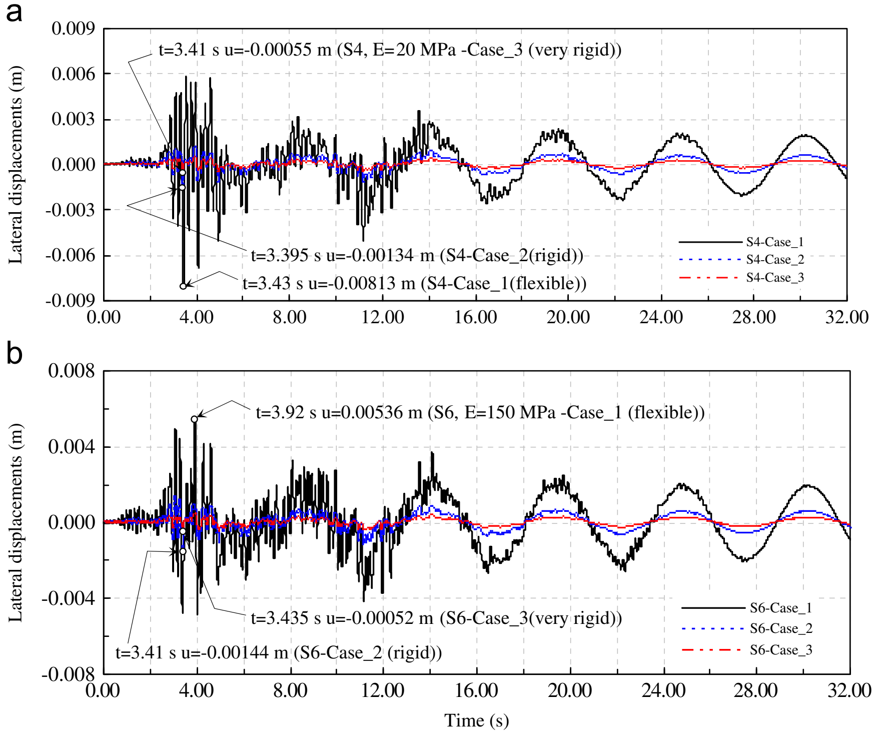

Fig. 14. Variations of the lateral displacements in time (a) for Case_1, Case_2 and Case_3 with S4 soil type and (b) for Case_1, Case_2 and Case_3 with S6 soil type.

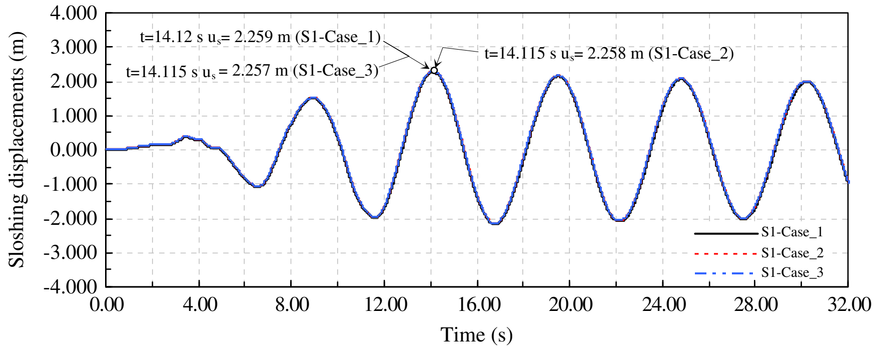

Fig. 15. Variations of the sloshing displacements in time for Case_1, Case_2 and Case_3 with S1 soil type.

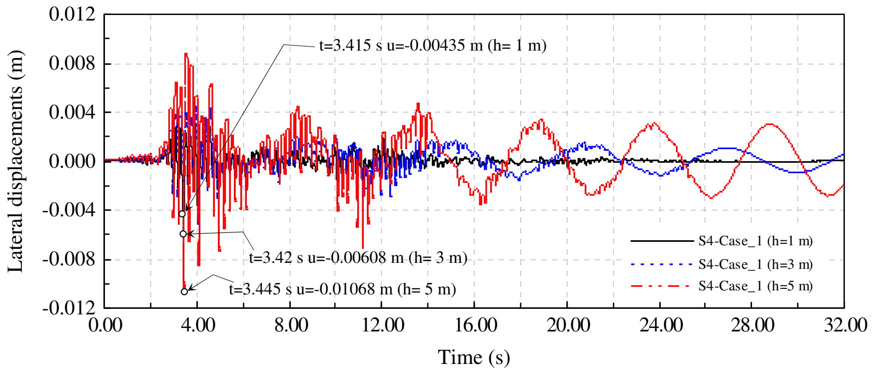

Fig. 16. Variations of the lateral displacements in time for Case_1(flexible) with S4 soil type considering different fluid levels.

comparison was made for S6 soil types in Fig. 14(b), the same pattern can be observed; the displacement response decreases as the rigidity of tank wall increases. For example, while the displacement response was calculated as 0.00536 m for Case_1, the same quantity was computed as 0.00144 m for Case_2, and this indicated that a response decrement of nearly 73% was obtained.

Similarly, the time histories of sloshing responses depending on the flexibility of tank wall are illustrated in Fig. 15. Based on the comparisons, one can conclude that the wall flexibility does not affect the sloshing response of the system so that all responses occurred as 2.26 m at 14.115–14.2 s, and the same deviations were found across time.

5.4. *Fluid interaction effects on the lateral displacement*

In order to evaluate the effects of fluid interaction, the calculated lateral displacements can be compared considering different fluid heights inside the container. Thus, the obtained responses and their occurrence times were comparatively presented in Table 4 and Fig. 16, taking into account five different fluid levels for Case_1 under the S4 backfill soil condition. The most important point concerning the comparison of the illustrations is that the increment in the fluid level notably increases the displacement response of the system; that is, while the lateral displacement is estimated as 0.00435 m for the fluid height of 1 m, the same quantity is computed as 0.01068 m for the fluid height of 5 m, and a dramatic increase of 145% takes place due to the fluid interaction. Therefore, the fluid effects must be definetely taken into consideration in design process for the tank stability.

6. Conclusions

A simple procedure is proposed for the seismic analysis of fluid-rectangular tank-backfill systems, coding a computer program. The procedure provides a rapid estimation of the seismic behaviour of the expressed systems, considering both fluid and backfill interaction effects. Analysis with this procedure requires significantly less computer memory capacity and shorter CPU times, compared to the numerical modelling typical of such interaction systems. To this end, the proposed analytical modeling and numerical simulations were performed for two real rectangular tank systems located in Bolu and Samsun provinces of Turkey.

Using modal analysis techniques, an efficient analytical procedure and finite element simulation for the backfill-rectangular tank–fluid interaction system were presented. The results of modal analyses using analytical and numerical solutions were shown to have acceptable accuracy compared with each other.

The lateral displacement responses change notably when both the backfill soil gets softer/stiffer and the wall flexibility varies. For flexible tanks, more specific variations were observed compared to the rigid tanks. Therefore, especially for flexible tanks, backfill interaction effects must be definetely considered in the design process. Similarly, fluid interaction must be also taken into account in design process for the tank stability. In this context, it can be easily stated that the rectangular tanks must not be designed only according to the typical projects because the local backfill soil conditions may change the system behavior.

The sloshing responses are not practically affected by the backfill interaction and wall flexibility in all cases. Therefore, the effects on the sloshing displacements can be ignored in the assessment of the seismic behavior of rectangular tanks.

One of the limitations of the proposed analytical solution stems from the assumption of complete bonding in the wall–soil interface. This can be rather inaccurate if the wall is very flexible, due to the development of unrealistic tensile stresses at the interface. However, this assumption was made in order to obtain a simplified model. On the other hand, the interface behavior can be taken into consideration by using special interface elements in the proposed finite element simulations. Within this framework, it seems that the analytical expressions and finite element simulations, when working together, may provide to engineers an effective method in the calculation of dynamic behavior and design of rectangular tanks.

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