Engineering Structures 116 (2016) 95–106

Contents lists available at ScienceDirect

Engineering Structures

journal homepage: www.elsevier.com/locate/engstruct

Effects of non-uniform temperature distribution on critical member temperature of steel tubular truss

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ARTICLE INFO

Article history:
Received 27 October 2015
Revised 19 February 2016
Accepted 24 February 2016

Keywords:
Circular Hollow Section (CHS)
Failure temperature
Localised fire
Multiple heated members
Non-uniform heating
Axially restrained compression member

ABSTRACT

This paper examines the effects of thermal restraint, caused by non-uniform temperature distribution in different members, on the failure temperatures of critical members of steel tubular trusses. Non-uniform temperature distribution develops in trusses exposed to localised fire attack. The truss member nearest to the fire source experiences the highest temperature, with reduced temperatures in the nearby members. The number of the nearby truss members being heated and their temperatures will affect the failure temperature of the critical truss member which has the highest temperature. The aim of this paper is to develop a simplified method to account for the effects of different numbers of members being simultaneously heated to different temperatures on the development of compression force and failure temperature of the critical member.

Finite Element (FE) simulations were carried out for Circular Hollow Section (CHS) trusses using the commercial Finite Element software ABAQUS v6.10-1 which has previously been validated by the authors. The simulation trusses were subjected to constant mechanical loads and then increasing temperatures until failure. The elevated temperature stress-strain curves were based on EN-1993-1-2 [1]. Initial geometrical imperfections were included, based on the lowest buckling mode from eigenvalue analysis.

The numerical study examined the effects of truss type, critical member slenderness, applied load ratio and axial restraint stiffness ratio on the failure temperatures of the critical truss members. The numerical simulation results were used to check the accuracy of a proposed simplified calculation method, combining linear elastic static truss analysis at ambient temperature and analytical equations to calculated the failure temperatures of thermally restrained truss members based on the regression equations of Wang et al. [2]. The calculation method was shown to be sufficiently accurate for fire resistant design purpose.

1. Introduction

Welded steel tubular trusses are frequently used to cover very large spaces, such as airports, exhibition halls, shopping malls and sport halls. For the fire resistant design of these large structures, the fire exposure is often assumed to be localised because of the small size of the fire compared to the large dimensions of the truss. Under localised fire, the different members of a truss will experience different temperatures. In welded trusses, because of restraint, non-uniform temperature distribution in different truss members will generate additional forces in the most heated member due to restrained thermal expansion.

Assessing the fire resistance of a steel truss exposed to localised fire involves quantifying the fire size, calculating the truss member temperatures and checking whether the critical member (the one with the highest temperature) has sufficient load carrying capacity. Quantification of localised fire can follow the method in EN-1991-1-2 [3], which calculates the size of the localised fire, including the height and temperature of the flame, as functions of the rate of heat release and distance to the fire source. Heat transfer analysis can then be used to obtain temperature distributions in the different members of the truss. Relevant research studies include Chen et al. [4,5] who tested and numerically modelled a steel roof truss without fire-proof coating under localised pool fire condition to obtain the truss temperature distributions and displacements. The members directly above the fire experienced the highest temperature, while the members away from the fire source experienced reduced temperatures. Whilst the quantification of localised fire behaviour and heat transfer analysis are important

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http://dx.doi.org/10.1016/j.engstruct.2016.02.044
0141-0296/

Nomenclature

The following symbols are used in this paper

A	cross section area of truss member
E	elastic (Young's) modulus
$F_{\beta l}$	function for the axial restraint stiffness
F_{ρ}	function for the initial axial load level
F_{λ}	function for the column slenderness
$F_{\text{cric},i}$	increase in tension force of the critical member when the ith adjacent member is heated
$F_{i,i}$	additional force in member "i"
$F_{\text{unit},i}$	change in internal force in the critical member when there is unit compressive force in member "i"
k_1	axial restraint stiffness at end 1
k_2	axial restrained stiffness at end 2
k_b	axial stiffness of member
$k_{c.0}$	axial stiffness of member at ambient temperature
k_f	modification factor for axial restraint ratio
k_i	axial restraint stiffness of member "i"
k_l	stiffness of the axial restraint
k_{total}	total axial restraint stiffness of the surrounding structure
l_b	length of member
n	total number of heated adjacent members
$P_{20\,^\circ\text{C}}$	member force at ambient temperature
$P_{\max}$	member force at buckling temperature from truss analysis
T_0	limiting temperature of the unrestrained member
$T_{20\,^\circ\text{C}}$	failure temperature from individual member analysis without axial restraint
T_f	failure temperature of member
$T_{\max}$	the highest temperature in the critical truss member
T_{θ}	failure temperature from individual member analysis with axial restraint
ρ_N	load ratio
λ	slenderness
β_l	axial restraint ratio
ε	engineering strain
ε_T	true strain
σ	engineering stress
σ_T	true stress
α_{th}	coefficient of thermal elongation of steel
ΔT_f	reduction in member failure temperature due to restrained thermal expansion
ΔT_{ABAQUS}	reduction in member failure temperature due to restrained thermal expansion, from ABAQUS simulation
$\Delta T_{\text{Wang et al.}}$	reduction in member failure temperature based on the regression equations of Wang et al. [2]
$\Delta T_{\max}$	temperature increase in the critical member
$\Delta F_{\text{single member}}$	increase in compression force of the critical member when one member is heated
$\Delta F_{\text{multiple members}}$	increase in compression force of the critical member when the adjacent members are heated

parts of fire resistant design, this paper will only focus on the mechanical behaviour of welded trusses with non-uniform temperature distributions in different members.

Due to complexity, the assessment of mechanical behaviour of non-uniformly heated truss is often resorted to using numerical modelling. For example, Lin et al. [6] carried out numerical simulations to investigate the effects of loading ratio, temperature distribution, fire location and size on the fire resistance of a steel roof truss under local fire exposure. Non-uniform temperature distribution along the truss was calculated by using the equations of Du and Li [7]. Yu et al. [8] simulated the behaviour of steel space structures under localised travelling fires. The same maximum temperature was assumed in the fire zone and the temperatures in the other zones decreased depending on the distance from the fire zone. Ho et al. [9] modelled an unprotected long span steel truss to examine both the temperature distribution along the steel truss and the effect of restrained thermal expansion under a moderate fire. They observed that large compressive forces were generated due to restrained thermal expansion under even small fires. Kotsovinos [10] attempted to recreate the failure mechanisms of the WTC towers and reached similar conclusions as others (e.g. Usmani et al. [11], FEMA report [12] and NIST report [13]). This study noted that additional forces were generated in the truss members as a result of restrained thermal expansion. This type of numerical modelling requires time and specialist expertise which many structural engineers do not possess. It is necessary to develop thorough understanding of the effects of non-uniform heating to develop a simplified calculation method that is easy to use by structural engineers without specialist training in detailed modelling of structural behaviour at elevated temperatures. This is the aim of the paper.

The key issue is calculating the changing force in the critical member of truss due to the effects of restrained thermal expansion. The effects of restrained thermal expansion on the behaviour and failure temperature of single steel member have been investigated by a number of researchers. For example, Wang and Moore [14,15] developed a general equation to calculate the additional compression force in steel column due to restrained thermal expansion. Ali et al. [16] tested 37 axially restrained steel columns in fire to investigate the influences of column slenderness ratio, axial restraint ratio and column load ratio on the failure temperatures of thermally restrained columns. Franssen [17] used SAFIR to numerically investigate the behaviour and failure temperatures of axially restrained columns at elevated temperatures. Wang [18] examined the post-buckling behaviour of axially restrained columns. An analytical method was derived to trace the entire column load-temperature relationship, including increasing axial compression due to restrained thermal expansion, initial buckling and post-buckling behaviour. Tan and Yuan [19] developed an analytical method to calculate fire resistances of non-uniformly heated columns. However, that method did not consider high slenderness and plasticity. Li et al. and Wang et al. [20,21] completed both experimental and numerical analyses on the response of restrained steel columns at elevated temperatures. Their findings are consistent with other research studies which have found that the failure temperatures of steel columns with high restraint ratio or high slenderness or small load ratio are higher than the buckling temperatures. Correia et al. [22] performed a parametric study on HEA, HEB and HEM steel columns with restrained thermal expansion. They provided a series of simplified equations to calculate the fire resistances and critical temperatures of restrained steel columns. In contrast to earlier findings, they noted that axial restraint had no effect on the fire resistance of steel columns. Wang et al. [2] carried out a regression analysis of their extensive numerical simulations and derived a set of analytical equations to calculate the restrained column buckling and failure temperatures. These analytical equations were derived based on numerical simulations of I-sections. These studies on axially restrained single members form important basis of knowledge for steel truss column under localised fire attack. However, there is a key difference: in steel trusses exposed to localised fires, a number of truss members are heated simultaneously. It is important to consider the interactions between the differently heated truss members.

The specific scope of this paper is to investigate the interactions between differently heated truss members due to non-uniform temperature distributions in trusses when subjected to localised fire attack and to develop a simplified method to calculate the failure temperature of the critical member (the most highly heated member). Since in any typical truss, the chord members have large section sizes relative to the brace members, the effect of thermal restraint on chord members is small. Therefore, this study will focus on the brace members of trusses. This work is based on parametric numerical simulations using the general purpose Finite Element software ABAQUS which has previously been validated by the authors [15].

2. Description of the Finite Element parametric study

2.1. Definition of failure (critical) temperature of axially restrained member in compression

Fig. 1 illustrates the general behaviour of an axially restrained steel member in compression, using its internal force – temperature relationship. Initially, the thermal elongation of the member is restrained and an additional compressive force is generated in the member (stage A). This results in the member experiencing temporary failure (point A). After this stage, the internal force in the member is relieved (stage B) and the member behaviour enters the post-buckling stage. Depending on the restraint stiffness, the member may still be able to sustain forces greater than the initial force at ambient temperature. For the purpose of this study, the member failure temperature is defined as the temperature at which the member force returns to the initial value at ambient temperature (point B). This definition has been used by Franssen [17], Wang [18], Correia et al. [22] and Wang et al. [2].

2.2. Parametric study cases

Fig. 2 shows the trusses used in the parametric study. In any typical truss, because the chord members have large section sizes relative to the brace members, the effect of thermal restraint on the chord members is small. Therefore, this study will focus on the brace members. In the numerical simulations, the truss is assumed to have continuous chord members and pin-joined brace members. The temperature distribution within each member is assumed to be uniform. Appendix A lists the detailed cross-section dimensions used in the parametric study.

In the parametric study, the temperature distributions in different brace members will be assumed, based on typical temperature distributions obtained by others. However, since the developed simplified method will be generally applicable, this assumption is not a problem.

The parametric study investigates the effects of the following parameters: truss type, load ratio, critical member slenderness and axial restraint ratio (defined in Eq. (4)). Table 1 lists the parametric study cases.

2.3. Material properties

The thermal expansion coefficient and non-linear elevated temperature engineering stress–strain curves were based on EN-1993-1-2 [1] as shown in Fig. 3. The ambient temperature elastic modulus of steel was assumed to be 210 GPa and the ambient temperature yield stress was assumed to be 355 N/mm^2. In the ABAQUS simulation model, the engineering stress–strain curve

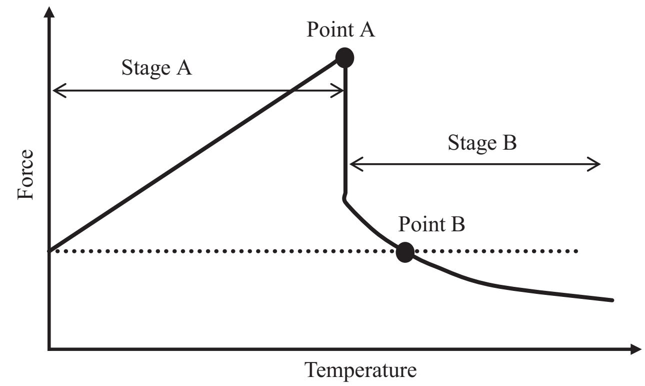

Fig. 1. Force – temperature behaviour of axially restrained compression member.

was converted into a true stress and logarithmic strain curve to consider the nonlinear effects of large displacements by using the following equations [23]:

$$\varepsilon_T = \ln(1+\varepsilon) \quad (1)$$

$$\sigma_T = \sigma \cdot (1+\varepsilon) \quad (2)$$

where ε_T, is the true strain; ε, is the engineering strain; σ_T, is the true stress; σ, is the engineering stress.

2.4. Finite Element type and initial imperfection

For the chord and brace members, ABAQUS element types S4R (4 noded shell element) or B21 (2 noded line element) may be used. Based on the author's previous findings [24], either using 2D line elements or 3D shell elements, is suitable for simulating the overall behaviour of welded tubular truss in fire. Therefore, to save computational cost, line elements were used.

Eigenvalue buckling analysis was performed first to define the possible buckling modes for the truss compressed members. Lanczos was chosen as eigensolver together with the requested five buckling modes [25]. Initial imperfections were included, based on the lowest buckling mode from eigenvalue analysis. The maximum initial imperfection was according to EN 1993-1-1 [26].

3. Validation of numerical model: comparison for single heated brace member in truss

When a single brace member in a truss is heated, it behaves as an axially restrained member with restrained thermal expansion. In analysing an axially restrained compression member, a spring is used to represent the restraint. In the truss, the restraint to the heated member comes from the surrounding structure. It is necessary to calculate the equivalent restraint stiffness. This section will compare the behaviour of singly heated brace members in trusses with representations of single members with attached axial spring.

Fig. 4 shows an isolated truss brace member, with the springs at both ends representing the surrounding structures.

This member can be represented by the member shown in Fig. 4 (b) with one spring stiffness, which is the model used in analysing axially restrained compressive members [9–13]. The equivalent one spring stiffness can be calculated as follows:

$$\frac{1}{k_{\text{total}}} = \frac{1}{k_1} + \frac{1}{k_2} \quad (3)$$

The equivalent one spring stiffness can be calculated based on static analysis of the truss. Take the truss shown in Fig. 5 as an example. The critical member is removed and replaced by equal

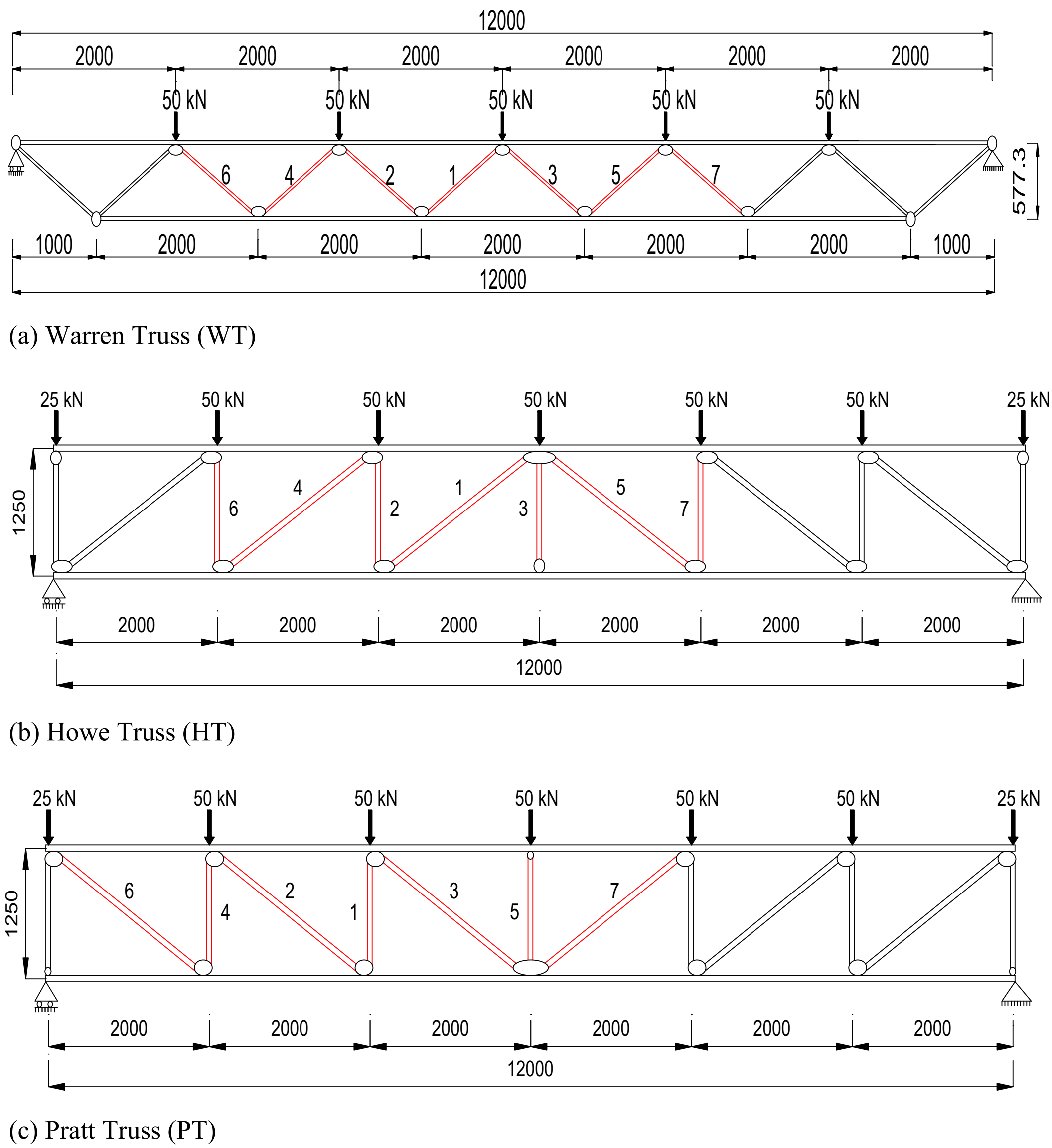

Fig. 2. Truss configurations (dimensions in mm).

Table 1
Simulation models with different parameters.

Simulation case	Applied load (kN)	Load ratio (ρ_N)	Slenderness (λ)	Axial restraint ratio (β_l)
WT1-M1	40	0.20	57	0.17
WT2-M1	90	0.45	57	0.17
WT3-M1	120	0.61	57	0.17
WT4-M1	75	0.47	57	0.12
WT5-M1	50	0.26	57	0.12
WT6-M1	75	0.68	84	0.08
WT7-M1	50	0.22	57	0.23
WT8-M1	75	0.47	84	0.24
WT9-M1	50	0.36	84	0.19
WT10-M1	100	0.65	44	0.14
WT11-M1	75	0.40	44	0.14
PT1-M5	100	0.40	49	0.01
PT2-M5	50	0.46	92	0.03
PT3-M1	75	0.60	33	0.05
HT1-M1	100	0.38	93	0.11

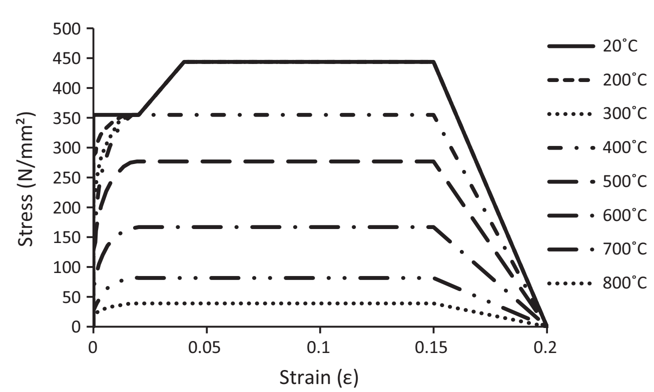

Fig. 3. Engineering stress–strain relationships of steel at elevated temperatures (according to EN 1993-1-2 [1]).

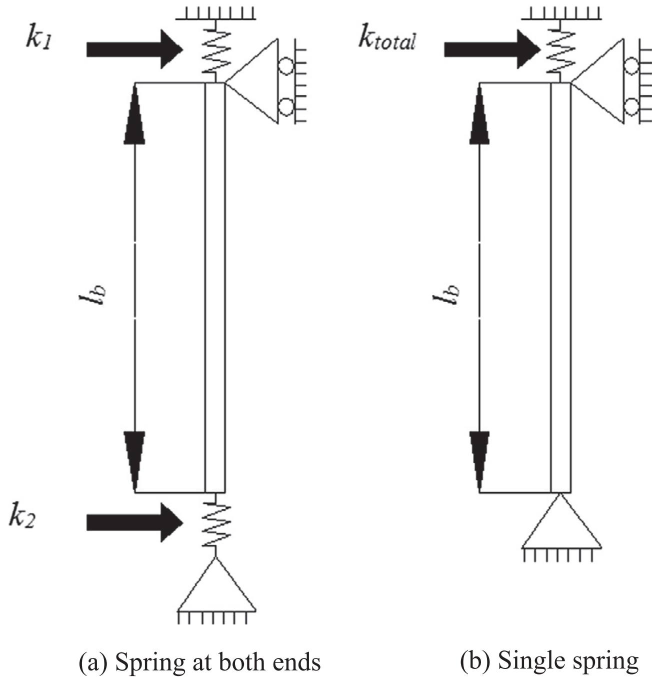

Fig. 4. Mechanical model for a restrained steel truss member.

compression force (P) applied at the two end joints. The overall stiffness k_{total} is simply P divided by the total separation of the two joints.

A number of trusses have been modelled, using both member analysis (Fig. 4(b)) and truss analysis where only one member is heated, for the trusses shown in Fig. 2.

In these simulations, different levels of load ratio, ρ_N, were considered by changing the applied load. The section sizes of both the top and bottom chord members were changed to vary the restraint stiffness.

Table 1 lists the comparison cases, for three different truss types, different load ratios, different critical member slendernesses and different axial restraint ratios. For identification, the name of each truss consists of the truss type (Warren (WT), Howe (HT) and Pratt (PT)), the truss number (such as WT1, WT2, and PT2) and the heated member number (e.g. M1 refers to member 1 being heated). For example, WT2-M1 refers to brace member 1 of Warren truss number 2 being heated.

The axial restraint ratio is expressed as follows:

$$\beta_l = \frac{k_{total}}{k_b} \tag{4}$$

$$k_b = \frac{EA}{l_b} \tag{5}$$

where k_b is the brace member stiffness.

Fig. 6 compares selective force – temperature curves between the member and truss based analyses, the vertical dashed line indicating the failure temperature. It is clear that the agreement is excellent throughout all the different phases of restrained truss member behaviour.

Table 2 compares the truss member failure temperatures between truss member analyses with or without axial restraint, and truss simulations. Results are given for both the load ratio ($P_\theta/P_{20\,^\circ C}$), which is the ratio of the member force at buckling temperature (P_{max}) from truss analysis to that at ambient temperature ($P_{20\,^\circ C}$), and member failure temperatures. Table 2 clearly shows that there is excellent agreement between the truss analysis results and the member analysis results with axial restraint. The member analysis results without restraint can give considerable overestimations of the restrained member failure temperatures.

This comparative study (both Fig. 6 and Table 2) clearly confirms that the single member based analysis, with axial restraint, is adequate to represent the behaviour of single heated truss member in truss. Also, it is important that the effects of axial restraints, in reducing the restrained truss member failure temperature, are included in fire resistant design.

4. Method of calculating failure temperature of axially restrained single compressive tubular member

Wang et al. [2] have conducted extensive numerical simulations of axially restrained single steel column at elevated temperatures and have proposed a method, based on regression analysis of their simulation results, to calculate the reductions in column failure temperatures from those without axial restraint. Their studies were for H-section columns. This section checks whether their calculation method is applicable to tubular members. For this check, numerical simulations were carried out for axially restrained members in compression with a range of load ratios, levels of axial restraint, and slendernesses. These three parameters govern the restrained column behaviour. Member 4 of the Warren truss (see in Fig. 2(a)) was considered as representation.

The calculation equations of Wang et al. [2] are:

$$T_f = T_0 - \Delta T_f \tag{6}$$

where T_0 is the limiting temperature of the unrestrained member; ΔT_f is the reduction in member failure temperature due to restrained thermal expansion.

$$\Delta T_f = F_{\beta_l} F_\rho F_\lambda \tag{7}$$

$$F_{\beta_l} = 12.432 - 12.796e^{-\beta_l/0.081} \tag{8}$$

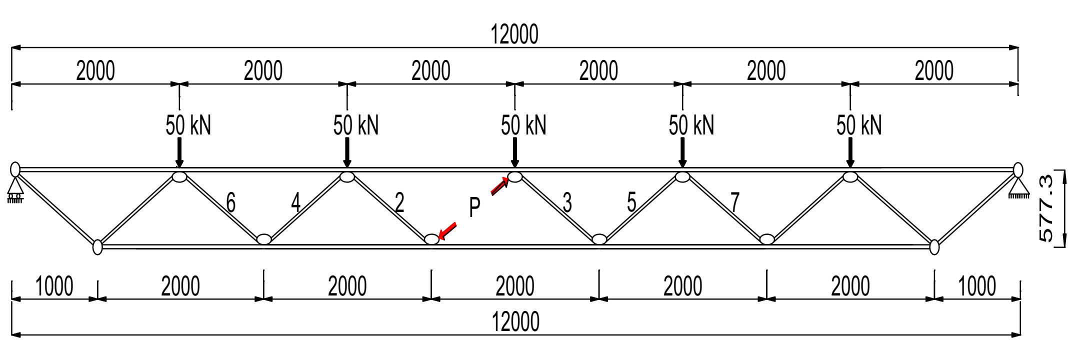

Fig. 5. Model to determine the restraint stiffness to heated member.

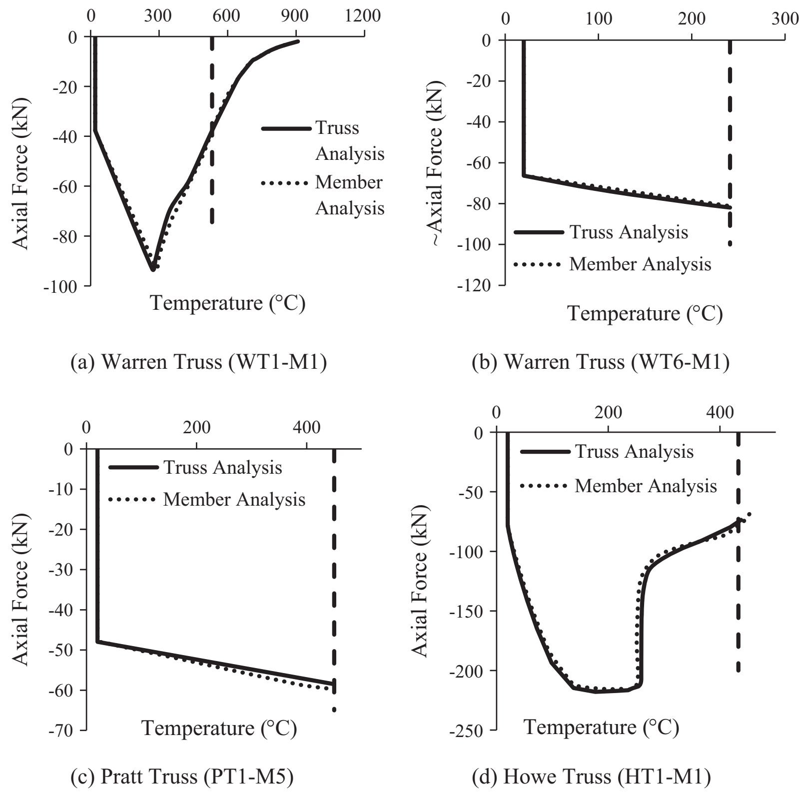

Fig. 6. Typical comparison of force – temperature curves between member and truss analyses.

$$F_\rho = 0.042 + 0.849\rho_N - 0.689\rho_N^2 + 0.204\rho_N^3 \quad (9)$$

$$F_\lambda = 28.624 + 1.053\lambda - 0.004\lambda^2 \quad (10)$$

where $F_{\beta l}$, F_ρ and F_λ represent the influences of axial restraint stiffness, initial axial load level and column slenderness respectively on ΔT_f.

The input parameters are defined as follows:

$$\beta_l = \frac{k_l}{k_{c.0}} \quad (11)$$

where β_l is the axial restraint stiffness ratio, ρ_N is the load ratio, λ is the member slenderness, k_l is the stiffness of axial restraint, $k_{c.0}$ is the axial stiffness of the member at ambient temperature $\left(k_{c.0} = \frac{EA}{l_b}\right)$.

4.1. Case 1: effect of load ratio, (ρ_N)

Table 3 compares the simulation results for different load ratios. In Table 3, $T_{20\,°C}$ and T_θ refer to the failure temperatures from individual member analysis without restraint (using the 20 °C member force) and with axial restraint respectively. ΔT_{ABAQUS} $(= T_{20\,°C} - T_\theta)$ is the reduction in member failure temperature due to restrained thermal expansion, from the ABAQUS simulation model. $\Delta T_{\text{Wang et al.}}$ is the reduction in failure temperature calculated according to the regression equations of Wang et al. [2]. For these members, the axial restraint ratio and the slenderness were 0.17 and 57 respectively.

The agreement between the ABAQUS results and the calculated results using the Wang et al. [2] equations is satisfactory. The discrepancy between ABAQUS modelling results and regression

Table 2
Comparison between failure temperatures of single heated truss member and change in member force.

Truss	Member analysis (without restraint) (°C)	Truss analysis (°C)	Member analysis (with restraint) (°C)	$P_{max}/P_{20\,°C}$
WT1-M1	688	532	533	2.44
WT2-M1	578	322	328	1.70
WT3-M1	526	231	235	1.35
WT4-M1	598	368	378	1.72
WT5-M1	657	510	520	2.10
WT6-M1	470	241	245	1.26
WT7-M1	679	546	549	3.81
WT8-M1	543	233	233	1.71
WT9-M1	586	345	347	2.00
WT10-M1	575	323	329	1.79
WT11-M1	620	470	477	1.54
PT1-M5	520	440	450	1.25
PT2-M5	571	315	320	1.99
PT3-M1	528	415	423	1.27
HT1-M1	644	418	433	2.80

Table 3
Reductions in member failure temperatures for different load ratios.

Load ratio (axial restraint ratio)	$T_{20\,°C}$ (°C)	T_θ (°C)	ΔT_{ABAQUS} (°C)	$\Delta T_{Wang\ et\ al.}$ (°C)
0.14 (0.17)	733	607	126	125
0.24 (0.17)	688	533	155	171
0.34 (0.17)	636	440	196	213
0.44 (0.17)	578	308	270	246
0.52 (0.17)	547	278	269	268
0.60 (0.17)	520	243	277	285

results for CHS sections is similar to that for H-section as originally observed in Wang et al. [2].

4.2. Case 2: effect of axial restraint ratio, (β_l)

For this case, two different load ratios, 0.24 and 0.55, were applied while the slenderness was kept constant at 57.

Table 4(a) and (b) presents comparisons of the results for these two load ratios.

The ABAQUS simulation results in Table 4(a) indicate that there is a cap in the reduction in restrained member failure temperature as the restraint stiffness increases. This finding correlates with the simulation results of Franssen [17]. This happens when the member force is relatively low and the restraint stiffness is high. The restrained member can resist further temperatures after initial failure at the maximum load and restrained members with different restraint stiffnesses behave very similarly during the post-buckling phase before the member forces return to the initial values at ambient temperature. This is shown in Fig. 7. The calculation equations of Wang et al. [2] do not predict this effect. Nevertheless, the rate of change in the reduction in failure temperatures slows down as the restraint stiffness increases. Furthermore, the calculation results using the Wang et al. [2] equations generally give higher reductions in member failure temperatures, therefore can be considered to be on the safe side.

4.3. Case 3: effect of slenderness, (λ)

For this case, different cross sections ($\Phi 76.1 \times 3$, $\Phi 60.3 \times 3$, $\Phi 48.3 \times 4$ and $\Phi 42.4 \times 4$) were used. The load ratio and the axial restraint stiffness were kept constant at values of 0.55 and 0.17 respectively and Table 5 compares the results. These results show very large reductions in member failure temperatures due to axial restraint and that the regression equations of Wang et al. [2] give reasonable predictions.

In summary, the failure temperatures of axially restrained tubular member in compression will be lower than that without thermal expansion. The reduction in member failure temperature depends on the load ratio, the axial restraint ratio and the slenderness of member. It is important that the restraint effect is considered in the fire safety design of trusses under localised heating. The method proposed by Wang et al. [2] for calculating the reduction in failure temperature of axially restrained column with H-section is suitable for tubular members in trusses.

Table 4
Comparison of failure temperatures of restrained tubular members (load ratio = 0.24).

Axial restraint ratio (β_l)	$T_{20\,°C}$ (°C)	T_θ (°C)	ΔT_{ABAQUS} (°C)	$\Delta T_{Wang\ etal.}$ (°C)
(a) Load ratio: ρ_N = 0.24				
0.02	688	654	34	39
0.08	688	540	148	122
0.12	688	540	148	151
0.17	688	540	148	173
0.30	688	540	148	192
(b) Load ratio ρ_N = 0.55				
0.02	537	507	30	61
0.08	537	316	221	194
0.12	537	293	244	238
0.17	537	264	273	271
0.30	537	241	296	302

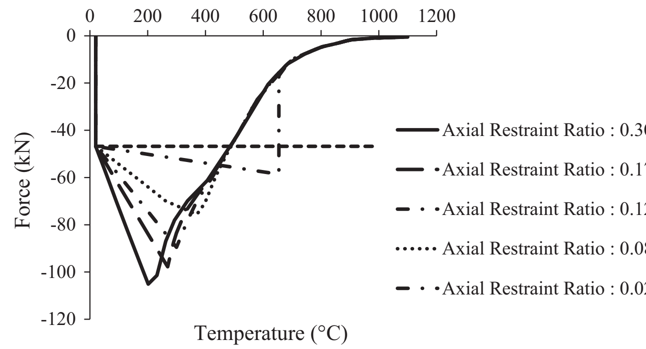

Fig. 7. Force – temperature curves of axially restrained members (load ratio: 0.24).

Table 5
Comparison of restrained member failure temperatures for different member slenderness.

Member slenderness (axial restraint ratio)	$T_{20\,°C}$ (°C)	T_θ (°C)	ΔT_{ABAQUS} (°C)	$\Delta T_{Wang\ et\ al.}$ (°C)
44 (0.17)	604	337	267	242
57 (0.17)	537	264	273	271
74 (0.17)	507	238	269	303
84 (0.17)	513	245	268	318

5. Effects on critical member with multiple truss members being heated

When a single truss member is heated, it is relatively easy to convert this member into an axially restrained compression member, as explained in the previous section. When additional members adjacent to the critical member are also heated under localised fire, the behaviour of the critical member is affected. The thermal elongations of the adjacent members will affect movements at the ends of the critical member, thereby affecting the additional compressive force generated in the critical member. This section develops a simplified method to quantify this change.

5.1. Assumed temperature distributions

Fig. 8 shows different temperature distributions. The truss dimensions are the same as those of WT4 in Fig. 2(a). The critical member is number 1, being exposed to the highest temperature, T_{max}. Fig. 8(a)–(d) illustrates heating only the critical member, heating the critical member plus the two adjacent members (one on each side), heating the critical member plus the four adjacent members (two on each side), and heating the critical member plus the six adjacent members (three on each side) respectively. The adjacent members are heated to $3/4T_{max}$ (members 2 and 3), $T_{max}/3$ (members 4 and 5) and $T_{max}/4$ (members 6 and 7) respectively, based on typical observations of others [4,6,7].

5.2. Effects of heating the adjacent members on the critical member

Fig. 9 compares the temperature–force curves for the different heating scenarios in Fig. 8 for the critical member (member 1). Two observations are immediately clear: (1) The rates of increase in critical member force are lower when multiple members are

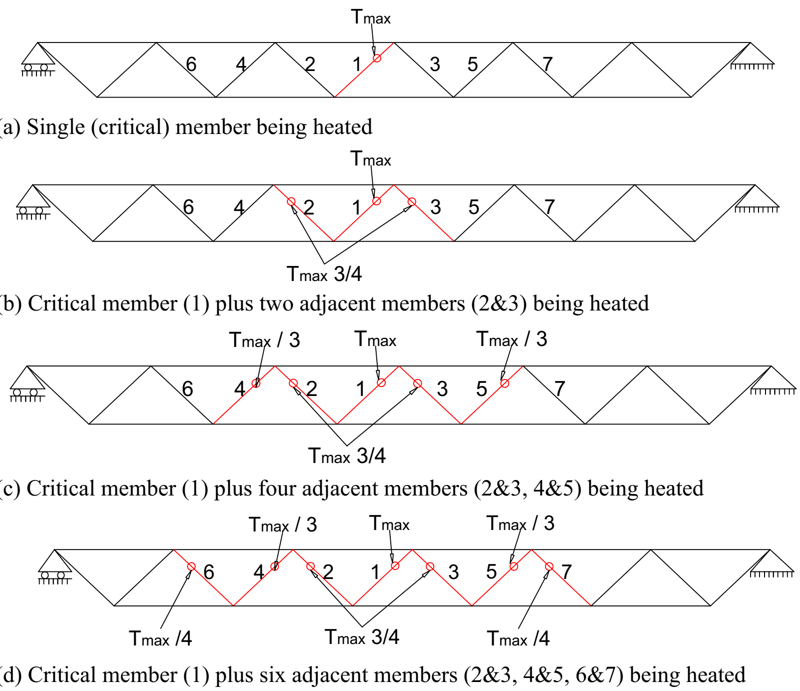

Fig. 8. Different scenarios of localised heating of a Warren truss.

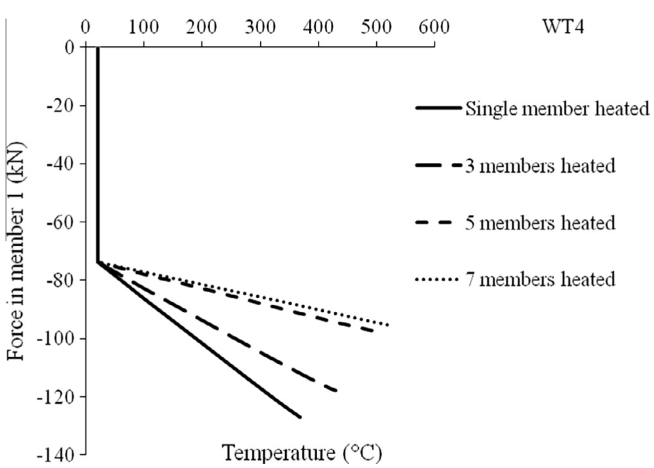

Fig. 9. Comparison of temperature – force curves for different numbers of members being heated.

heated than when only the critical member is heated. Therefore, it would be on the safe side when considering only the critical member being heated in fire resistant design. However, the calculation results would be too conservative. (2) There is negligible effect on the failure temperature of the critical member when the adjacent heated members are three or more members away from the critical member. This is indicated by the closeness of the results in Fig. 9 between heating 5 members (heated members being 2 members away from the critical member) and 7 members (heated members being 3 members away from the critical member). Therefore, it is only necessary to consider up to two adjacent members on each side of the critical member.

The reduction in the rate of force increase in the critical member due to the adjacent members being heated may be explained by referring to Fig. 10. When the adjacent members next to the critical member are heated, the joints at both ends of the critical member (Joints A and B in Fig. 10) are pushed away from each other, thus relieving the thermal elongation of the critical member, thereby reducing the unrestrained thermal expansion of the critical member.

5.3. Method for analysis and design

The results in Fig. 9 indicate that the effect of heating the adjacent members is to reduce the rate of compression force increase in the critical member. This is equivalent to reducing the restraint stiffness. This section will develop a simplified method to calculate this reduction.

The general equation for calculating the increase in compression force of a member due to axial restraint is (13):

$$\Delta F = \frac{k_{\text{total}} k_b}{k_{\text{total}} + k_b}\left(\alpha_{\text{th}} \Delta T_{\max} l_b - \frac{F}{AE_T} l_b\right) \tag{12}$$

Because the purpose of this simplification is to derive an equivalent restraint stiffness, it is only necessary to consider the initial stage behaviour of the restrained member. This means that the member mechanical strain change due to changing mechanical properties (the second part in brackets in Eq. (12)) is negligible. If only one single member is heated, the initial rate of increase in the compression force of member can be calculated using Eq. (13).

$$\Delta F_{\text{single member}} = \frac{k_{\text{total}} k_b}{k_{\text{total}} + k_b} \alpha_{\text{th}} \Delta T_{\max} l_b \tag{13}$$

where α_{th} is the coefficient of thermal elongation of steel and $\Delta T_{\max}$ is the temperature increase in the critical member.

When the adjacent members are heated, the increase in the compression force of critical member is given by:

$$\Delta F_{\text{multiple members}} = \frac{k_{\text{total}} k_b}{k_{\text{total}} + k_b} \alpha_{\text{th}} \Delta T_{\max} l_b - \sum_{i=1}^{n} F_{\text{cric},i} \tag{14}$$

where $F_{\text{cric},i}$ is the increase in tension force of the critical member when the ith adjacent member is heated to a temperature corresponding to a rise in temperature of $\Delta T_{\max}$ in the critical member and n is the total number of heated adjacent members.

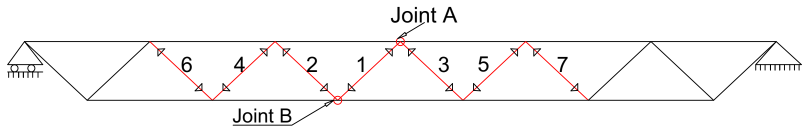

Fig. 10. Effects of adjacent members being heated on the critical member.

Using the format of Eq. (13) for only the critical member being heated, the equation for calculating the increase in compression force for multiple members being heated can be written as:

$$\Delta F_{\text{multiple members}} = k_f \frac{k_{\text{total}} k_b}{k_{\text{total}} + k_b} \alpha_{\text{th}} \Delta T_{\max} l_b \tag{15}$$

where the multiplication factor k_f is defined as:

$$k_f = \frac{\Delta F_{\text{multiple members}}}{\Delta F_{\text{single member}}} \tag{16}$$

Therefore the equivalent restraint stiffness ratio for multiple members being heated can be calculated as:

$$\beta_l = \frac{k_{\text{total}}}{\frac{k_b + k_{\text{total}}}{k_f} - k_{\text{total}}} \tag{17}$$

Table 6
Comparison of critical member failure temperatures between analytical results and numerical simulation results.

Truss ID	Number of heated members, and modification factor for axial restraint ratio	ABAQUS member analysis without axial restraint (°C)	Analytical method (°C)	ABAQUS truss analysis (°C)
WT1-M1	Member 1, $k_f = 1.00$	688	535	532
WT1-M3	Members 1–3, $k_f = 0.73$		552	563
WT1-M5	Members 1–5, $k_f = 0.46$		582	597
WT1-M7	Members 1–7, $k_f = 0.44$		585	597
WT2-M1	Member 1, $k_f = 1.00$	578	332	322
WT2-M3	Members 1–3, $k_f = 0.74$		358	362
WT2-M5	Members 1–5, $k_f = 0.45$		409	415
WT2-M	Members 1–7, $k_f = 0.44$		412	415
WT4-M1	Member 1, $k_f = 1.00$	598	375	368
WT4-M3	Members 1–3, $k_f = 0.74$		407	428
WT4-M5	Members 1–5, $k_f = 0.45$		462	507
WT4-M7	Members 1–7, $k_f = 0.41$		470	519
WT5-M1	Member 1, $k_f = 1.00$	657	499	510
WT5-M3	Members 1–3, $k_f = 0.74$		522	532
WT5-M5	Members 1–5, $k_f = 0.46$		558	555
WT5-M7	Members 1–7, $k_f = 0.45$		560	556
WT6-M1	Member 1, $k_f = 1.00$	470	222	241
WT6-M3	Members 1–3, $k_f = 0.73$		269	276
WT6-M5	Members 1–5, $k_f = 0.47$		328	378
WT6-M7	Members 1–7, $k_f = 0.45$		333	380
WT7-M1	Member 1, $k_f = 1.00$	679	504	546
WT7-M3	Members 1–3, $k_f = 0.75$		516	562
WT7-M5	Members 1–5, $k_f = 0.48$		542	585
WT7-M7	Members 1–7, $k_f = 0.46$		545	585
WT8-M1	Member 1, $k_f = 1.00$	543	219	233
WT8-M3	Members 1–3, $k_f = 0.74$		243	267
WT8-M5	Members 1–5, $k_f = 0.53$		274	326
WT8-M7	Members 1–7, $k_f = 0.53$		274	325
WT10-M1	Member 1, $k_f = 1.00$	575	329	323
WT10-M3	Members 1–3, $k_f = 0.73$		366	346
WT10-M5	Members 1–5, $k_f = 0.39$		440	424
WT10-M7	Members 1–7, $k_f = 0.37$		446	436
WT11-M1	Member 1, $k_f = 1.00$	620	428	470
WT11-M3	Members 1–3, $k_f = 0.72$		453	493
WT11-M5	Members 1–5, $k_f = 0.43$		500	526
WT11-M7	Members 1–7, $k_f = 0.42$		501	530
PT1-M1	Member 1, $k_f = 1.00$	610	569	583
PT1-M3	Members 1–3, $k_f = 0.10$		610	614
PT2-M1	Member 1, $k_f = 1.00$	544	443	387
PT2-M3	Members 1–3, $k_f = 0.14$		536	537
PT3-M1	Member 1, $k_f = 1.00$	528	415	451
PT3-M3	Members 1–3, $k_f = 0.47$		469	492
PT3-M5	Members 1–5, $k_f = 0.20$		505	517
HT1-M1	Member 1, $k_f = 1.00$	644	412	418
HT1-M3	Members 1–3, $k_f = 0.93$		421	461
HT1-M5	Members 1–5, $k_f = 0.93$		421	461
HT1-M7	Members 1–7, $k_f = 0.93$		421	461

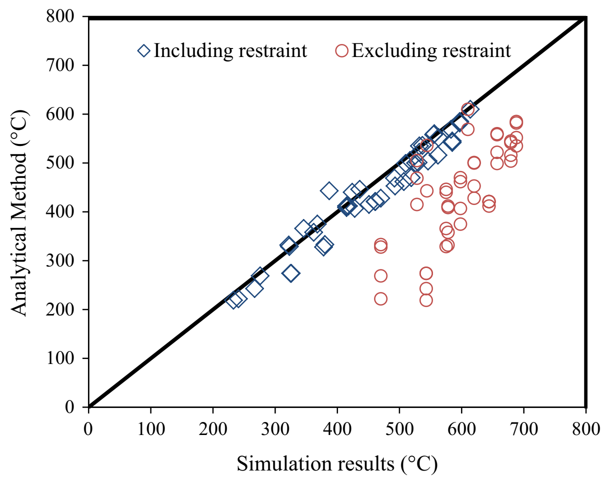

Fig. 11. Comparison for failure temperatures of critical members between analytical calculations including restraint, analytical calculations not including restraint and ABAQUS simulation results.

Table 7
Additional compression forces in members 1, 2, 3, 4, 5 when one single member is heated to the temperature corresponding to ΔT_{max} = 100 °C, and comparison with ABAQUS simulation results.

Member no.	Eq. (13) (N)	ABAQUS (N)
Member 1 ($F_{1,1}$)	12,612	12,635
Members 2 and 3 ($F_{2,2}$ and $F_{3,3}$)	8601	8646
Member 4 ($F_{4,4}$)	2146	2177
Member 5 ($F_{5,5}$)	2057	2076
Members 6 and 7 ($F_{6,6}$ and $F_{7,7}$)	769	801

This restraint stiffness ratio should be used in Eq. (6) when calculating the reduction in failure temperature of the critical member for the case of multiple heated members.

As explained in the introduction section, the purpose of this paper is to develop a simplified calculation method that may be used by structural engineers that do not have specialist training in modelling structures at elevated temperature. Therefore, only linear static truss analysis at ambient temperature will be considered for calculating force $F_{cric,i}$ in Eq. (14). This force change in the critical member is caused by the additional force ($F_{i,i}$) in member "i" when its own thermal expansion is restrained. $F_{i,i}$ can be calculated by using Eq. (13), but substituting the stiffness terms associated with the critical member by those associated with member i, and also the temperature increase in member "i" corresponds to the temperature increase of ΔT_{max} in the critical member. In the ambient temperature linear elastic static truss analysis, this is carried out simply by replacing member "i" with force $F_{i,i}$. The resulting force in the critical member is the sought after value $F_{crit,i}$. It should be pointed out that because it is the rate of force change at increasing temperature that is required, a nominal value (say 1 °C) for ΔT_{max} can be used.

5.4. Comparison of failure temperature between numerical and analytical results

The accuracy of the proposed analytical method has been checked for the different trusses used in the parametric study presented in Section 3. Table 6 compares the failure temperatures of trusses under different fire scenarios between ABAQUS truss analysis and analytical solution by using the equivalent restraint stiffness ratio in the regression equations of Wang et al. [2]. The results of member analysis without axial restraint are also included in

Table 8
Changes in the critical member force due to heating the adjacent members.

Heated member no.	Force in the critical member (member 1) (N, tension positive)
2 ($F_{cric,2}$)	301
3 ($F_{cric,3}$)	2924
4 ($F_{cric,4}$)	1888
5 ($F_{cric,5}$)	1763
6 ($F_{cric,6}$)	250
7 ($F_{cric,7}$)	263

Table 6. Not including the effects of axial restraint grossly overestimate the truss member failure temperatures and is unsafe. Including the effects of heating three or more members adjacent to the critical member (members 1–7 in the table) has very little influence compared to heating only the two adjacent members (Members 1–5 in the table). For clarity, Fig. 11 compares the ABAQUS truss analysis results with the analytical results. The results in Table 6 (reproduced in Fig. 11) indicate that the proposed analytical method can give reasonably close predictions compared to the ABAQUS truss simulation results with the calculation results generally on the safe side.

6. An example

An example is provided in this section to illustrate application of the procedure to calculate the failure temperature of the critical member in a truss with multiple heated members.

Referring to Fig. 8(d), which shows the temperature distribution of members closest to the fire source, WT4 truss was chosen for this example. Calculate the failure temperature of the critical member (member 1, the member with the highest temperature).

Step 1:

Replace each of the members being heated by a unit compressive force (1 kN) and carry out linear elastic static truss analysis to obtain the relative movement of the two ends of the member. The restrained stiffness of the members can be determined as follows:

k_1 = 12,307 N/mm, k_2 = 12,500 N/mm, k_3 = 12,307 N/mm, k_4 = 12,403 N/mm, k_5 = 12,500 N/mm, k_6 = 19,210 N/mm, k_7 = 12,403 N/mm.

In the same step, note the changes in internal force in the critical member. These are the force coefficients to calculate the force in the critical member when there is a unit compressive force in any of the heated members. These values (compression positive, tension negative) are:

$F_{Unit,2}$ = 35 N/1 kN in member 2, $F_{Unit,3}$ = 340 N/1 kN in member 3, $F_{Unit,4}$ = 880 N/1 kN in member 4, $F_{Unit,5}$ = 587 N/1 kN in member 5, $F_{Unit,6}$ = 325 N/1 kN in member 6, $F_{Unit,7}$ = 342 N/1 kN in member 7.

Step 2:

Assume the critical member temperature increases by 100 °C. The actual temperature value is not important because the stiffness modification factor will be independent of this temperature. Use Eq. (13) to calculate the additional compression force ($F_{i,i}$) in each of the heated members, the temperature increases in the other members being according to the temperature distribution in Fig. 8(d) corresponding to ΔT_{max} = 100 °C in the critical member (member 1). Table 7 shows the analytical results and compares the analytical results with the ABAQUS simulation results, demonstrating good agreement.

Step 3:

Multiplying the forces in Table 7 by the respective force coefficients of step 1 gives the force changes in the critical member (member 1) when the adjacent members are heated to their

Table 9
Final compression forces in the critical member with different adjacent members being heated, and resulting stiffness modification factors.

Members being heated	Compression force (Eq. (14)) (N)	Stiffness modification factor (Eq. (16)) (k_f)
Member 1	12,612	1.00
Members 1, 2 and 3	9387	0.74
Members 1, 2, 3, 4 and 5	5735	0.45
Members 1, 2, 3, 4, 5, 6 and 7	5222	0.41

Table 10
Comparison of the critical member failure temperatures for different numbers of the adjacent members being heated.

Heated members	Member analysis without restraint (°C)	ABAQUS truss analysis (°C)	Analytical method (°C)
Member 1	598	368	375
Members 1–3		428	407
Members 1–5		507	462
Members 1–7		519	470

respective temperatures corresponding to $\Delta T_{max} = 100$ °C. These values are listed in Table 8.

Step 4:

Use Eq. (14) to calculate the total increase in the internal compression force of the critical member. The total increase in the compression force can then be used in Eq. (16) to calculate the restraint stiffness modification factor. Table 9 summarises the calculation results.

Step 5:

Use the regression equations of Wang et al. [2] to calculate the reduction in failure temperature of the critical member. The calculation results are presented in Table 10. Also shown in Table 10 are the critical member failure temperatures without considering restraint and from ABAQUS truss simulation.

7. Conclusions

This paper has presented the results of a numerical investigation of the behaviour of welded steel tubular trusses under non-uniform temperature distributions in different truss members. The simulations were carried out for Circular Hollow Section (CHS) trusses using the commercial Finite Element software, ABAQUS v6.10-1 [25]. Based on the numerical simulation results, a simplified method, combining linear elastic static truss analysis with analytical equations, has been developed to calculate the failure temperatures of critical members (the members with the highest temperatures) in non-uniformly heated trusses.

Based on the study, the following conclusions can be drawn:

(1) It is important to consider the effects of restrained thermal expansion when trusses are non-uniformly heated. Not including this effect in fire safety design could grossly overestimate failure temperatures of the critical truss members.
(2) The behaviour of a single heated brace member in truss can be represented by an isolated member with axial restraint. The regression equations of Wang et al. [2], originally developed for axially restrained columns with H cross-section, are applicable to tubular members.

Table A.1
Dimensions of truss members.

Truss	Member type	Dimensions (mm)
WT1, WT2 and WT3	Bottom and top chords	Φ219.1 × 12
	Outer bracings	Φ168.3 × 5
	Inner bracings (member 1, 2, 3 and 5)	Φ60.3 × 3
WT4	Bottom and top chords	Φ168.3 × 10
	Outer bracings	Φ168.3 × 5
	Inner bracings (member 1, 2, 3 and 5)	Φ60.3 × 3
WT5	Bottom and top chords	Φ193.7 × 12
	Outer bracings	Φ168.3 × 5
	Inner bracings (member 1, 2, 3 and 5)	Φ60.3 × 3
WT6	Bottom and top chords	Φ168.3 × 10
	Outer bracings	Φ168.3 × 5
	Inner bracings (member 1, 2, 3 and 5)	Φ42.4 × 4
WT7	Bottom and top chords	Φ244.5 × 12
	Outer bracings	Φ168.3 × 5
	Inner bracings (member 1, 2, 3 and 5)	Φ60.3 × 3
WT8	Bottom and top chords	Φ244.5 × 12
	Outer bracings	Φ168.3 × 5
	Inner bracings (member 1, 2, 3 and 5)	Φ42.4 × 4
WT9	Bottom and top chords	Φ219.1 × 12
	Outer bracings	Φ168.3 × 5
	Inner bracings (member 1, 2, 3 and 5)	Φ42.4 × 4
WT10 and WT11	Bottom and top chords	Φ168.3 × 10
	Outer bracings	Φ168.3 × 5
	Inner bracings (member 1, 2, 3 and 5)	Φ76.1 × 3
PT1	Bottom and top chords	Φ193.7 × 10
	Compression brace members (member 4, 6 and 8)	Φ114.3 × 5
	Tension brace members (member 5, 7 and 9)	Φ168.3 × 6.3
	Middle brace member (member 10)	Φ76.1 × 2.5
PT2	Bottom and top chords	Φ193.7 × 10
	Compression brace members (member 4, 6 and 8)	Φ76.1 × 4
	Tension brace members (member 5, 7 and 9)	Φ76.1 × 6.3
	Middle brace member (member 10)	Φ42.4 × 4
PT3	Bottom and top chords	Φ193.7 × 10
	Brace members	Φ76.1 × 2.5
HT1	Bottom and top chords	Φ193.7 × 10
	Outer bracings (member 5, 6 and 7)	Φ168.3 × 6.3
	Inner bracings (member 4, 8 and 9)	Φ76.1 × 5

(3) When the critical member and some of the adjacent members in a truss are heated, heating the adjacent members alleviates the effects of restrained thermal expansion of the critical member: the increase in compression force of the critical member is smaller when under multiple member heating than when under single member heating.
(4) Static truss analyses at ambient temperature can be performed to obtain the changes in force in the critical member due to heating the adjacent members. Superposition can be used to obtain the total change in force in the critical member by summing up all the force changes in the critical member due to heating the adjacent members.
(5) It is not necessary to consider the effects of on the critical member if the heated member is three or more members away from the critical member. This assumption is on the safe side.
(6) An equivalent restraint stiffness can be calculated based on the total force change in the critical member to account for multiple members being heated. Using this equivalent restraint stiffness in the regression equations of Wang et al. [2], the failure temperature of the critical member can be calculated. Comparisons between the analytical results and the numerical simulation results have confirmed that the analytical method is on the safe side and sufficiently accurate for design.

Appendix A

See Table A.1.

References

[1] CEN. Design of steel structures. In: Part, editor. EN 1993-1-2-structural fire design. London: British Standard Institute; 2005.
[2] Wang P, Wang YC, Li GQ. A new design method for calculating critical temperatures of restrained steel column in fire. Fire Saf J 2010;45:349–60.
[3] CEN. Actions on structures. Part: EN 1991-1-2: general actions – actions on structures exposed to fire. British Standard; 2002.
[4] Chen C, Zhang W. Structural behaviors of steel roof truss exposed to pool fire. J Central South Univ 2012;19:2054–60.
[5] Chen C-k, Zhang W. Comparative experimental investigation on steel staggered-truss constructed with different joints in fire. J Constr Steel Res 2012;77:43–53.
[6] Lin S-M, Du Y, Fu J-X. Design strategies of fire safety for steel roof trusses exposed to localised fire. In: 8th International conference on structures in fire, Shanghai, China; 2014. p. 163–70.
[7] Du Y, Li GQ. A new temperature–time curve for fire-resistance analysis of structures. Fire Saf J 2012;54:113–20.
[8] Yu HX, Sun XQ, Wong K. Resistance of steel space frames subjected to localized traveling fire. In: 8th International conference on structures in fire, Shanghai, China; 2014. p. 1121–8.
[9] Ho H, Chung K, Wong Y. Structural fire engineering study on unprotected long span steel trusses. Procedia Eng 2011;14:1132–9.
[10] Kotsovinos P. Analysis of the structural response of tall buildings under multifloor and travelling fires. University of Edinburgh; 2013.
[11] Usmani A, Chung Y, Torero JL. How did the WTC towers collapse: a new theory. Fire Saf J 2003;38:501–33.
[12] Quintiere JG, Di Marzo M, Becker R. A suggested cause of the fire-induced collapse of the world trade towers. Fire Saf J 2002;37:707–16.
[13] Gann R. NIST NCSTAR 1: final report of the national construction safety team on the collapse of the world trade center twin towers. Gaithersburg, MD: National Institute of Standards and Technology (NIST); 2005.
[14] Wang YC, Moore DB. The effect of frame continuity on the critical temperature of steel columns. In: Third kenrensky conference on flobal trends in structural engineering, Singapore; 1994. p. 681–6.
[15] Wang YC, Moore DB. Effect of thermal restraint on column behaviour in a frame. In: Kashiwagi T, editor. 4th International symposium on fire safety science. Ottawa: IAFSS; 1994. p. 1055–66.
[16] Ali FA, Shepherd P, Randall M, Simms IW, O'Connor DJ, Burgess I. The effect of axial restraint on the fire resistance of steel columns. J Constr Steel Res 1998;46:305–6.
[17] Franssen J-M. Failure temperature of a system comprising a restrained column submitted to fire. Fire Saf J 2000;34:191–207.
[18] Wang YC. Postbuckling behavior of axially restrained and axially loaded steel columns under fire conditions. J Struct Eng 2004;130:371–80.
[19] Tan K, Yuan W. Buckling of elastically restrained steel columns under longitudinal non-uniform temperature distribution. J Constr Steel Res 2008;64:51–61.
[20] Li G-Q, Wang P, Wang Y. Behaviour and design of restrained steel column in fire, Part 1: fire test. J Constr Steel Res 2010;66:1138–47.
[21] Wang P, Wang Y, Li G-Q. Behaviour and design of restrained steel column in fire: Part 2. Parameter study. J Constr Steel Res 2010;66:1148–54.
[22] Correia AJM, Rodrigues JPC, Gomes FC. A simplified calculation method for fire design of steel columns with restrained thermal elongation. Comput Struct 2013;116:20–34.
[23] Boresi AP, Schmidt RJ. Advanced mechanics of materials. 6th ed. John Wiley and Sons; 2003.
[24] Ozyurt E, Wang YC. Effects of truss behaviour on critical temperatures of welded steel tubular truss members exposed to uniform fire. Eng Struct 2015;88:225–40.
[25] ABAQUS/Standard. In: Ed, editor. K a S Hibbitt. Version 6.10-1 ed. USA: K. a. S. Hibbit; 2010.
[26] CEN. Design of steel structures. In: Part: editor. EN 1993-1-1-general rules and rules for building. London: British Standard Institute; 2005.