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Effects of discretization schemes on numerical results of natural convection heat transfer

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Abstract. Two dimensional natural convection heat transfer in a square enclosure is numerically analyzed. Laminar natural-convection heat transfer is studied by solving the Navier-Stokes and energy equations using finite volume method. Numerical solutions of flow field are obtained by Semi-Implicit Method for Pressure-Linked Equations (SIMPLE). Effects of the discretization schemes on numerical solutions are compared with each other.

Keywords: Natural convection, Navier-Stokes equations, Discretization scheme
PACS: 02.60.-x, 02.70.Dh, 04.25.-g

INTRODUCTION

Natural convection heat transfer can be used in modeling of various engineering problems such as heating and cooling of buildings, energy storage, electronics cooling, meteorological events, nuclear reactor designs, etc. Number of numerical studies about natural convection heat transfer have increased with the development of computer technology. Many engineering problems were solved by finite difference, finite element and finite volume methods. Two dimensional natural convection problem was solved by de Vahl Davis [1] using alternating direction implicit (ADI) algorithm. Some of the applications of buoyancy induced flow in cavities were investigated by Ostrach [2]. Solution procedures of the finite volume method were explained extensively by Patankar [3] and Versteeg and Malalasekera [4]. Shi and Khodadadi [10] studied laminar natural convection heat transfer in a differentially heated square enclosure thin fin mounted on the hot wall using SIMPLE algorithm and finite volume discretization method. Laminar natural convection heat transfer in square enclosures with half active and half insulated vertical walls was numerically investigated by Valencia and Frederick [5]. A SIMPLE like algorithm on co-located grid system for convective heat transfer has been developed by Wang et all [6]. Mustafa [7] studied the effects of vertical parabolic walls on natural convection heat transfer in a parabolic enclosure with the Rayleigh number ranging from 10^3 to 10^6. Raos [8] investigated laminar natural convection phenomena in enclosed spaces using SIMPLE iterative procedure and finite volume method. Natural convection around a tilted heated square cylinder kept in an enclosure has been studied by De and Dalal [9].

In the present paper, we analyze numerically two dimensional natural convection heat transfer in a square enclosure. Using finite volume method, laminar natural-convection heat transfer is investigated by solving the Navier-Stokes and energy equations. Numerical solutions of flow field are obtained by SIMPLE.

MATHEMATICAL FORMULATION

The natural convection is considered to be two-dimensional, steady and laminar. Thermophysical properties of the fluid assumed to be constant except the density. The buoyancy effects upon momentum transfer are taken into account through the Boussinesq approximation. The governing equations for the steady natural convection flow using conservation of mass, momentum and energy can be written as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$\rho u\frac{\partial u}{\partial x}+\rho v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right), \tag{2}$$

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$$\rho u\frac{\partial v}{\partial x}+\rho v\frac{\partial v}{\partial y}=-\frac{\partial p}{\partial y}+\rho g\beta(T-T_0)+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right), \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right). \tag{4}$$

The non-dimensional parameters taking place in equations (1-4) are defined as

$$\begin{aligned} &X=\tfrac{x}{L}, Y=\tfrac{y}{L}, U=\tfrac{uL}{\alpha}, V=\tfrac{vL}{\alpha}, \theta=\tfrac{T-T_c}{T_h-T_c},\\ &P=\tfrac{pL^2}{\rho\alpha^2}, \mathrm{Pr}=\tfrac{v}{\alpha}, Ra=\tfrac{\rho g\beta(T_h-T_c)L^3}{\mu\alpha}. \end{aligned} \tag{5}$$

The governing equations in dimensionless form are obtained as

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0, \tag{6}$$

$$U\frac{\partial U}{\partial x}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\mathrm{Pr}\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right), \tag{7}$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+Ra\,\mathrm{Pr}\,\theta+\mathrm{Pr}\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right), \tag{8}$$

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}. \tag{9}$$

The system of equations (6-9) was solved numerically employing a finite control volume procedure based on SIMPLE algorithm given by Patankar [3].

The general form of momentum and energy equations with general variable ϕ, which represents diffusion flux, heat flux, etc. is

$$\nabla\cdot\left(\rho\vec{U}\phi\right)=\nabla\cdot(\Gamma\nabla\phi)+S, \tag{10}$$

where $\vec{U}=U$ for one dimensional region and $\vec{U}=(U,V)$ for two dimensional region.

Equation (10) can be written in one dimensional region as

$$\frac{\partial}{\partial x}(\rho U\phi)=\frac{\partial}{\partial x}\left(\Gamma\frac{\partial\phi}{\partial x}\right)+S. \tag{11}$$

For two dimensional region, we have

$$\frac{\partial}{\partial x}(\rho U\phi)+\frac{\partial}{\partial y}(\rho V\phi)=\frac{\partial}{\partial x}\left(\Gamma\frac{\partial\phi}{\partial x}\right)+\frac{\partial}{\partial y}\left(\Gamma\frac{\partial\phi}{\partial y}\right)+S. \tag{12}$$

In this form of the equation, general variable ϕ, diffusion flux Γ, source term S are chosen from Table 1.

TABLE 1. Definitions of coefficients to the governing equations

ϕ	Γ	S
U	Pr	$-\frac{\partial P}{\partial X}$
V	Pr	$-\frac{\partial P}{\partial Y}+Ra\,\mathrm{Pr}\,\theta$
θ	1	1

The numerical solution of Navier-Stokes differential equations was performed by staggered grid. Using the differential transformation technique, the governing differential equations become basic algebraic equations. The domain has been discretized using a regular Cartesian mesh. This discretization process can be done by various methods, for example, central difference scheme, first order upwind scheme, hybrid scheme, power law scheme. The algebraic form of the control volume discretization is

$$a_P\phi_P = a_E\phi_E + a_W\phi_W + a_N\phi_N + a_S\phi_S + b. \tag{13}$$

Coefficients of algebraic equations are

$$\begin{aligned} a_E &= D_eA(|Pe_e|) + max[-F_e,0],\ a_w = D_WA(|Pe_w|) + max[F_w,0],\\ a_N &= D_nA(|Pe_n|)\ +\ max[-F_n,0],\ a_S = D_sA(|Pe_s|)\ +\ max[F_s,0],\\ a_P &= a_E + a_W + a_N + a_S + F_e - F_w + F_n - F_s. \end{aligned}$$

The convective mass flux per unit area F is given below for all directions

$$F_e = u_eA_e,\ F_w = u_wA_w,\ F_n = u_nA_n,\ F_S = u_sA_s.$$

D is the diffusion conductance of control volume,

$$D_e = \frac{\mathrm{Pr}A_e}{(\delta x)_e},\ D_w = \frac{\mathrm{Pr}A_w}{(\delta x)_w},\ D_n = \frac{\mathrm{Pr}A_n}{(\delta x)_n},\ D_s = \frac{\mathrm{Pr}A_s}{(\delta x)_s}.$$

Peclet number is defined as $Pe = F/D$ and $A(|Pe|)$ functions are chosen from Table 2.

TABLE 2. $A(|Pe|)$ function

Discretization scheme	$A(\lvert Pe\rvert)$ function
Central Difference	$1-0.5\lvert Pe\rvert$
Upwind	1
Hybrid	$[1-0.5\lvert Pe\rvert]$
Power law	$\left[1-0.1\lvert Pe\rvert^5\right]$

Distributions of velocity and pressure area are shown in Figure 1. U and V velocities are defined on the control surface. Peclet numbers are calculated from the velocity component.

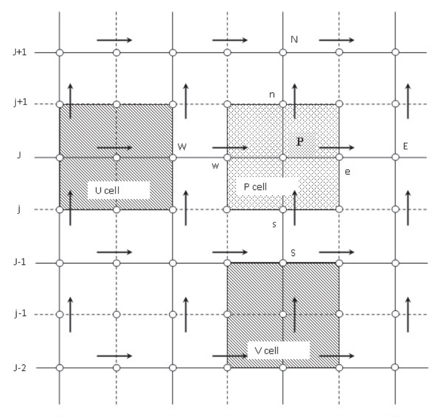

FIGURE 1. Two dimensional control volume

Source term S linearized and selected from Table 1 according to the discretization method. Linearized source term is given as

$$S = S_C + S_P\phi_P.$$

Prandtl number is 0.71 for air. Numerical calculations are performed for Rayleigh number ranging from 10^4 to 10^6 for square enclosure. Numerical solutions of the equations have been solved using a FORTRAN computer code. After

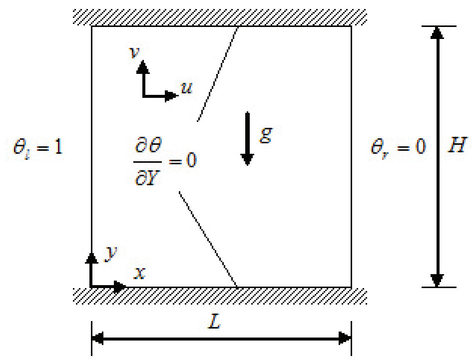

FIGURE 2. Two dimensional square enclosure and boundary conditions

a grid refinement study, all the calculations were performed with a 120x120 grid. The boundary conditions of square enclosure are given in Figure 2.

The local and average Nusselt number along the left wall for numerical solutions are given as follows

$$Nu_L = \left.\frac{\partial \theta}{\partial X}\right|_{X=0} \text{ and } \overline{Nu_L} = \frac{1}{H}\int_0^1 Nu_L dY.$$

The numerical results of natural convection for $Ra = 10^6$ are given in Figure 3.

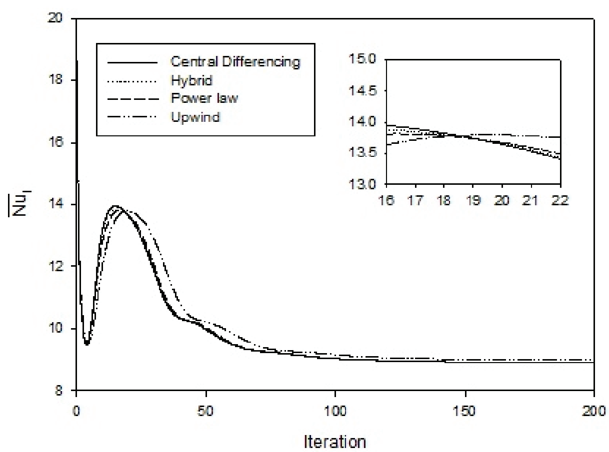

FIGURE 3. Average Nusselt numbers for different discretization schemes

CONCLUSION

Two dimensional natural convection heat transfer in a square enclosure is numerically analyzed. Laminar natural-convection heat transfer is studied by solving the Navier-Stokes and energy equations using finite volume method. Numerical solutions of flow field are obtained by SIMPLE method. Effects of the discretization schemes on numerical solutions are compared with each other.

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